2.3 Pulse-Propagation Equation

- The study of most nonlinear effects in optical fibers involves the use of **short pulses** with widths ranging from **~10 ns** to **10 fs**.
- Both **dispersive** and **nonlinear effects** influence their **shapes** and **spectra**.
- By using 2.1 Maxwell's Equations, it can be written in the form

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}_{\mathrm{NL}}}{\partial t^2}, \qquad (2.3.1)$$

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

$$\mathbf{P}(\mathbf{r},t) = \mathbf{P}_L(\mathbf{r},t) + \mathbf{P}_{NL}(\mathbf{r},t)$$

$$\nabla\times\nabla\times\mathbf{E} \equiv \nabla(\nabla\cdot\mathbf{E}) - \nabla^2\mathbf{E} = -\nabla^2\mathbf{E}$$

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}_{NL}}{\partial t^2}$$

- The **electric field** can be written in the form by **slowly varying envelope approximation** (separate the **rapidly varying part**)

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{2}\hat{x}[E(\mathbf{r},t)\exp(-i\omega_0 t) + \text{c.c.}], \quad (2.3.2)$$

  – **E(r, *t*) : slowly varying function of time** (relative to **the optical period**).

- The **polarization components** $\mathbf{P}_L$ and $\mathbf{P}_{\mathrm{NL}}$ can also be expressed in a similar way by writing

$$\mathbf{P}_L(\mathbf{r},t) = \frac{1}{2}\hat{x}[P_L(\mathbf{r},t)\exp(-i\omega_0 t) + \text{c.c.}], \quad (2.3.3)$$

$$\mathbf{P}_{\mathrm{NL}}(\mathbf{r},t) = \frac{1}{2}\hat{x}[P_{\mathrm{NL}}(\mathbf{r},t)\exp(-i\omega_0 t) + \text{c.c.}]. \quad (2.3.4)$$

➤ The **linear component P_L** can be obtained by substituting **Eq. (2.3.3)** in **(2.1.9)**

$$P_L(\mathbf{r},t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi_{xx}^{(1)}(t-t')E(\mathbf{r},t')\exp[i\omega_0(t-t')]\,dt'$$
$$= \frac{\varepsilon_0}{2\pi}\int_{-\infty}^{\infty} \tilde{\chi}_{xx}^{(1)}(\omega)\tilde{E}(\mathbf{r},\omega-\omega_0)\exp[-i(\omega-\omega_0)t]\,d\omega, \qquad (2.3.5)$$

$\tilde{E}$ (r, ω): Fourier transform of E(r, t)

$$\tilde{\mathrm{E}}(\mathbf{r},\omega) = \int_{-\infty}^{\infty} \mathrm{E}(\mathbf{r},t)\exp(i\omega t)\,dt. \qquad (2.1.12)$$

$$\mathbf{P}_L(\mathbf{r},t) = \tfrac{1}{2}\hat{x}[P_L(\mathbf{r},t)\exp(-i\omega_0 t) + \text{c.c.}], \qquad (2.3.3)$$

$$\mathbf{P}_L(\mathbf{r},t) = \varepsilon_0 \int_{-\infty}^{t} \chi^{(1)}(t-t')\cdot\mathbf{E}(\mathbf{r},t')\,dt', \qquad (2.1.9)$$

$$\mathbf{E}(\mathbf{r},t) = \tfrac{1}{2}\hat{x}[E(\mathbf{r},t)\exp(-i\omega_0 t) + \text{c.c.}], \qquad (2.3.2)$$

In considering Eq. (2.1.9) and Eq. (2.3.2)

- The **nonlinear component** $P_{NL}(r, t)$ is obtained by **substituting Eq. (2.3.4)** in **Eq. (2.1.10)**

$$\mathbf{P}_{NL}(\mathbf{r},t) = \frac{1}{2}\hat{x}[P_{NL}(\mathbf{r},t)\exp(-i\omega_0 t) + \text{c.c.}]. \qquad (2.3.4)$$

$$\mathbf{P}_{NL}(\mathbf{r},t) = \varepsilon_0 \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t} dt_2 \int_{-\infty}^{t} dt_3 \times \chi^{(3)}(t-t_1, t-t_2, t-t_3) \vdots \mathbf{E}(\mathbf{r},t_1)\mathbf{E}(\mathbf{r},t_2)\mathbf{E}(\mathbf{r},t_3). \qquad (2.1.10)$$

- The **time dependence** of $\chi^{(3)}$ in Eq. (2.1.10) is given by the product of three **delta functions** of the form $\delta(t - t_1)$ (if the **nonlinear response** is assumed to be **instantaneous**)

$$\mathbf{P}_{NL}(\mathbf{r},t) = \varepsilon_0 \chi^{(3)} \vdots \mathbf{E}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t). \qquad (2.3.6)$$

- The assumption of **instantaneous nonlinear response** amounts to **neglecting** the contribution of **molecular vibrations** to $\chi^{(3)}$ (the Raman effect).

- Both **electrons** and **nuclei** respond to the **optical field** in a **nonlinear manner**.
- The **nuclei response** is **inherently slower** compared with the **electronic response**.
- For **silica fibers**, the **vibrational** or **Raman response** occurs over a time scale **60–70 fs**.
- Thus, Eq. (2.3.6) is approximately valid for **pulse widths** >1 ps.

$$\mathbf{P}_{\mathrm{NL}}(\mathbf{r},t) = \varepsilon_0 \chi^{(3)} \vdots \mathbf{E}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t). \tag{2.3.6}$$

- When **Eq. (2.3.2)** is substituted in **Eq. (2.3.6)**, $\mathbf{P}_{\mathrm{NL}}(\mathbf{r}, t)$ is found to have a term oscillating at ω_0 and another term oscillating at the third-harmonic frequency $3\omega_0$.

$$\mathbf{E}(\mathbf{r},t) = \tfrac{1}{2}\hat{x}[E(\mathbf{r},t)\exp(-i\omega_0 t) + \text{c.c.}], \qquad (2.3.2)$$

$$\mathbf{P}_{\mathrm{NL}}(\mathbf{r},t) = \varepsilon_0 \chi^{(3)} \vdots \mathbf{E}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t)\mathbf{E}(\mathbf{r},t). \qquad (2.3.6)$$

- The **latter term** requires **phase matching** and is generally **negligible** in optical fibers.

- To obtain the **wave equation** for the **slowly varying amplitude** $E(\mathrm{r}, t)$, it is **more convenient** to work in the **Fourier domain**.
- This is generally **not possible** as Eq. (2.3.1) is **nonlinear** because of the **intensity dependence** of $\varepsilon_{\mathrm{NL}}$.

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}_{\mathrm{NL}}}{\partial t^2}, \qquad (2.3.1)$$

- In one **approach**, $\varepsilon_{\mathrm{NL}}$ is treated as a **constant** during the derivation of the **propagation equation**.
- The **approach** is **justified in view of** the
  - **slowly varying envelope approximation**
  - **perturbative nature** of P_{NL}.

$$\varepsilon_{\mathrm{NL}} = \frac{3}{4}\chi^{(3)}_{xxxx}|E(\mathbf{r},t)|^2. \qquad (2.3.8)$$

- Substituting Eqs. (2.3.2) through (2.3.4) in Eq. (2.3.1), the **Fourier transform** $\tilde{E}(r,\omega-\omega_0)$, defined as

$$\tilde{E}(\mathbf{r},\omega-\omega_0)=\int_{-\infty}^{\infty}E(\mathbf{r},t)\exp[i(\omega-\omega_0)t]\,dt, \qquad (2.3.9)$$

is found to satisfy the **Helmholtz equation (*$k_0 = \omega/c$*)**

$$\mathbf{E}(\mathbf{r},t)=\tfrac{1}{2}\hat{x}[E(\mathbf{r},t)\exp(-i\omega_0 t)+\text{c.c.}], \qquad (2.3.2)$$

$$\mathbf{P}_{\text{NL}}(\mathbf{r},t)=\tfrac{1}{2}\hat{x}[P_{\text{NL}}(\mathbf{r},t)\exp(-i\omega_0 t)+\text{c.c.}]. \qquad (2.3.4)$$

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}=\mu_0\frac{\partial^2\mathbf{P}_L}{\partial t^2}+\mu_0\frac{\partial^2\mathbf{P}_{\text{NL}}}{\partial t^2}, \qquad (2.3.1)$$

- Substituting Eqs. (2.3.2) through (2.3.4) in Eq. (2.3.1), the **Fourier transform** $\tilde{E}\,(r,\omega-\omega_0)$, defined as

$$\tilde{E}(\mathbf{r},\omega-\omega_0)=\int_{-\infty}^{\infty}E(\mathbf{r},t)\exp[i(\omega-\omega_0)t]\,dt, \qquad (2.3.9)$$

is found to satisfy the **Helmholtz equation (*$k_0 = \omega/c$*)**

$$\nabla^2\tilde{E}+\varepsilon(\omega)k_0^2\tilde{E}=0, \qquad (2.3.10)$$

the **dielectric constant**

$$\varepsilon(\omega)=1+\tilde{\chi}_{xx}^{(1)}(\omega)+\varepsilon_{NL} \qquad (2.3.11)$$

nonlinear part *ε_{NL}*

$$\varepsilon_{NL}=\frac{3}{4}\chi_{xxxx}^{(3)}|E(\mathbf{r},t)|^2. \qquad (2.3.8)$$

- The **dielectric constant** can be used to define the **refractive index** $\tilde{n}$ and the **absorption coefficient** $\tilde{\alpha}$.
- Both $\tilde{n}$ and $\tilde{\alpha}$ become **intensity dependent** because of $\boldsymbol{\varepsilon}_{\mathbf{NL}}$. It is customary to introduce

$$\tilde{n} = n + n_2|E|^2, \qquad \tilde{\alpha} = \alpha + \alpha_2|E|^2 \qquad (2.3.12)$$

- Using $\boldsymbol{\varepsilon = (\tilde{n} + i\,\tilde{\alpha}/2k_0)^2}$ and Eqs. (2.3.8) and (2.3.11), the **nonlinear-index coefficient** n_2 and the **two-photon absorption coefficient** $\boldsymbol{\alpha_2}$ are given by

$$n_2 = \frac{3}{8n}\mathrm{Re}(\chi^{(3)}_{xxxx}), \qquad \alpha_2 = \frac{3\omega_0}{4nc}\mathrm{Im}(\chi^{(3)}_{xxxx}). \qquad (2.3.13)$$

$$\varepsilon_{NL} = \frac{3}{4}\chi^{(3)}_{xxxx}|E(\mathbf{r},t)|^2. \qquad (2.3.8)$$

$$\varepsilon(\omega) = 1 + \tilde{\chi}^{(1)}_{xx}(\omega) + \varepsilon_{NL} \qquad (2.3.11)$$

- The **linear index** $\boldsymbol{n}$ and the **absorption coefficient** $\boldsymbol{\alpha}$ are related to the **real** and **imaginary parts** of $\tilde{\boldsymbol{x}}_{\boldsymbol{xx}}^{(1)}$ as in Eqs. (2.1.15) and (2.1.16).

$$n(\omega) = 1 + \tfrac{1}{2}\mathrm{Re}[\tilde{\chi}^{(1)}(\omega)], \tag{2.1.15}$$

$$\alpha(\omega) = \frac{\omega}{nc}\mathrm{Im}[\tilde{\chi}^{(1)}(\omega)], \tag{2.1.16}$$

- As $\boldsymbol{\alpha_2}$ is **relatively small** for silica fibers, it is often **ignored**.

- Equation (2.3.10) (**Helmholtz equation**) can be solved by using the method of **separation of variables**. If we assume a solution of the form

$$\tilde{E}(r,\omega-\omega_0)=F(x,y)\tilde{A}(z,\omega-\omega_0)\exp(i\beta_0 z), \qquad (2.3.14)$$

$\tilde{A}$ **(z,ω)**: slowly varying function of **z**

$\boldsymbol{\beta_0}$: wave number

- Helmholtz Eq. (2.3.10) leads to the following two equations for ***F*(*x*, *y*)** and $\tilde{A}$**(*z*, *ω*):**

$$\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}+\left[\varepsilon(\omega)k_0^2-\tilde{\beta}^2\right]F=0, \qquad (2.3.15)$$

$$2i\beta_0\frac{\partial\tilde{A}}{\partial z}+(\tilde{\beta}^2-\beta_0^2)\tilde{A}=0 \qquad (2.3.16)$$

$$\nabla^2\tilde{E}+\varepsilon(\omega)k_0^2\tilde{E}=0, \qquad (2.3.10)$$

- In obtaining **Eq. (2.3.16)**, the **second derivative $\partial^2 \tilde{A} / \partial z^2$** was neglected since $\tilde{A}$ **(z, ω)** is assumed to be a **slowly varying function** of **z**.

$$2i\beta_0 \frac{\partial \tilde{A}}{\partial z} + (\tilde{\beta}^2 - \beta_0^2)\tilde{A} = 0. \qquad (2.3.16)$$

- The **wave number** $\tilde{\beta}$ is determined by solving the **eigenvalue equation (2.3.15)** for the **fiber modes** using a procedure similar to that used in Section 2.2.

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + [\varepsilon(\omega)k_0^2 - \tilde{\beta}^2]F = 0, \qquad (2.3.15)$$

- The **dielectric constant** $\boldsymbol{\varepsilon(\omega)}$ in **Eq. (2.3.15)** can be approximated by

$$\varepsilon = (n + \Delta n)^2 \approx n^2 + 2n\Delta n, \qquad (2.3.17)$$

  - $\boldsymbol{\Delta n}$: small perturbation

$$\Delta n = n_2 |E|^2 + \frac{i\tilde{\alpha}}{2k_0}. \qquad (2.3.18)$$

- **Eq. (2.3.15)** can be solved using **first-order perturbation theory**. We first replace $\boldsymbol{\varepsilon}$ with $\boldsymbol{n^2}$ and obtain the modal distribution $\boldsymbol{F(x,y)}$, and the corresponding wave number $\boldsymbol{\beta(\omega)}$.

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + [\varepsilon(\omega)k_0^2 - \tilde{\beta}^2]F = 0, \qquad (2.3.15)$$

- For a **single-mode fiber**, ***F*(*x*,*y*)** corresponds to **the modal distribution** of the fundamental fiber mode **HE_{11}** given by **Eqs. (2.2.12)** and **(2.2.13)**, or by the **Gaussian approximation (2.2.14)**.

$$F(x,y) \approx \exp[-(x^2+y^2)/w^2], \qquad (2.2.14)$$

- We then include the effect of **Δ*n*** in **Eq. (2.3.15).**
- In **the first-order perturbation theory**, **Δ*n*** does not affect the **modal distribution** *F*(*x*,*y*).
- The **eigenvalue $\tilde{\beta}$** becomes

$$\tilde{\beta}(\omega) = \beta(\omega) + \Delta\beta(\omega), \qquad (2.3.19)$$

Where

$$\Delta\beta(\omega) = \frac{\omega^2 n(\omega)}{c^2 \beta(\omega)} \frac{\iint_{-\infty}^{\infty} \Delta n(\omega) |F(x,y)|^2 dxdy}{\iint_{-\infty}^{\infty} |F(x,y)|^2 dxdy} \qquad (2.3.20)$$

- This step completes the **formal solution** of **Eq. (2.3.1)** to the first order in perturbation $\mathbf{P}_{\mathbf{NL}}$.

$$\nabla^2 \mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_L}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}_{\mathrm{NL}}}{\partial t^2}, \qquad (2.3.1)$$

- Using **Eqs. (2.3.2)** and **(2.3.14)**, the **electric field E(r, *t*)** can be written as :

$$E(r,t) = \frac{1}{2}\hat{x}\{F(x,y)A(z,t)\exp[i(\beta_0 z - \omega_0 t)] + c.c.\}, \qquad (2.3.21)$$

A(*z*, *t*) : the slowly varying **pulse envelope**

$$\mathbf{E}(\mathbf{r},t) = \tfrac{1}{2}\hat{x}[E(\mathbf{r},t)\exp(-i\omega_0 t) + \mathrm{c.c.}], \qquad (2.3.2)$$

$$\tilde{E}(\mathbf{r},\omega-\omega_0) = F(x,y)\tilde{A}(z,\omega-\omega_0)\exp(i\beta_0 z), \qquad (2.3.14)$$

- The **Fourier transform** $\tilde{A}$ **(*z*,*ω* −*ω*₀)** of **A(z, *t*)** satisfies **Eq. (2.3.16)**, which can be written as

$$\frac{\partial \tilde{A}}{\partial z} = i[\beta(\omega) + \Delta\beta(\omega) - \beta_0]\tilde{A}, \qquad (2.3.22)$$

we used **Eq. (2.3.19)** $\tilde{\beta}(\omega) = \beta(\omega) + \Delta\beta(\omega)$,

and approximated $\tilde{\beta}^2 - \beta_0^2$ by $2\beta_0(\tilde{\beta} - \beta_0)$.

- The **physical meaning** of this equation is clear.
- Each **spectral component** within the **pulse envelope** acquires a **phase shift** whose magnitude is both **frequency** and **intensity dependent** as it propagates down the fiber, .

$$2i\beta_0 \frac{\partial \tilde{A}}{\partial z} + (\tilde{\beta}^2 - \beta_0^2)\tilde{A} = 0. \qquad (2.3.16)$$

$$\tilde{\beta}(\omega) = \beta(\omega) + \Delta\beta(\omega), \qquad (2.3.19)$$

- At this point, we can go back to the **time domain** by taking the **inverse Fourier transform** of **Eq. (2.3.22)**, and obtain the propagation equation for **A(*z*, *t*)**.
- As an **exact functional form** of $\boldsymbol{\beta(\omega)}$ is rarely known, it is useful to expand $\boldsymbol{\beta(\omega)}$ in a **Taylor series** around the carrier frequency $\boldsymbol{\omega_0}$ *as*

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2 \beta_2 + \frac{1}{6}(\omega - \omega_0)^3 \beta_3 + \cdots, \qquad (2.3.23)$$

where $\boldsymbol{\beta_0 \equiv \beta(\omega_0)}$ and other parameters are defined as

$$\beta_m = \left(\frac{d^m \beta}{d\omega^m}\right)_{\omega=\omega_0} \qquad (m = 1, 2, \cdots). \qquad (2.3.24)$$

- A similar expansion should be made for $\boldsymbol{\Delta\beta(\omega)}$**,** i.e.,

$$\Delta\beta(\omega) = \Delta\beta_0 + (\omega - \omega_0)\Delta\beta_1 + \frac{1}{2}(\omega - \omega_0)^2 \Delta\beta_2 + \cdots, \qquad (2.3.25)$$

$\boldsymbol{\Delta\beta_m}$ is defined similar to **Eq. (2.3.24).**

- The **cubic** and **higher-order terms** in the expansion **(2.3.23)** are negligible if the spectral width of the pulse satisfies the condition $\boldsymbol{\Delta\omega \ll \omega_0}$.

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2 \beta_2 + \frac{1}{6}(\omega - \omega_0)^3 \beta_3 + \cdots, \qquad (2.3.23)$$

- Their neglect is consistent with **the quasi-monochromatic assumption** used in the derivation of **Eq. (2.3.22)**.
- If $\boldsymbol{\beta_2 \approx 0}$ for some specific values of $\boldsymbol{\omega_0}$, it may be necessary to include the $\boldsymbol{\beta_3}$ term.
- Under the same conditions, we can use the approximation $\boldsymbol{\Delta\beta \approx \Delta\beta_0}$ in **Eq. (2.3.25)**.

$$\Delta\beta(\omega) = \Delta\beta_0 + (\omega - \omega_0)\Delta\beta_1 + \frac{1}{2}(\omega - \omega_0)^2 \Delta\beta_2 + \cdots, \qquad (2.3.25)$$

- After these simplifications in **Eq. (2.3.22)**, we take the **inverse Fourier transform** using

$$\frac{\partial \tilde{A}}{\partial z} = i[\beta(\omega) + \Delta\beta(\omega) - \beta_0]\tilde{A}, \qquad (2.3.22)$$

$$A(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{A}(z,\omega-\omega_0)\exp[-i(\omega-\omega_0)t]d\omega. \qquad (2.3.26)$$

- During the **Fourier-transform operation**, $\boldsymbol{\omega - \omega_0}$ is replaced by the differential operator $\boldsymbol{i(\partial/\partial t)}$.
- The resulting equation for $\boldsymbol{A(z, t)}$ becomes

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = i\Delta\beta_0 A. \qquad (2.3.27)$$

- The $\boldsymbol{\Delta\beta_0}$ term on the right side of **Eq. (2.3.27)** includes the effects of **fiber loss** and **nonlinearity**

- Using $\beta(\omega) \approx n(\omega)\omega/c$ and assuming that $F(x,y)$ in **Eq. (2.3.20)** does not vary much over the pulse bandwidth, **Eq. (2.3.27)** takes the form

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = ir(\omega_0)|A|^2 A, \qquad (2.3.28)$$

- Where the **nonlinear parameter** γ is defined as

$$\gamma(\omega_0) = \frac{n_2(\omega_0)\omega_0}{cA_{eff}}. \qquad (2.3.29)$$

$$\Delta\beta(\omega) = \frac{\omega^2 n(\omega)}{c^2 \beta(\omega)} \frac{\iint_{-\infty}^{\infty} \Delta n(\omega)|F(x,y)|^2\,dxdy}{\iint_{-\infty}^{\infty} |F(x,y)|^2\,dxdy}. \qquad (2.3.20)$$

$$\Delta n = n_2|E|^2 + \frac{i\tilde{\alpha}}{2k_0}. \qquad (2.3.18)$$

- In obtaining **Eq. (2.3.28)** the **pulse amplitude *A*** is assumed to be **normalized** such that $|A|^2$ represents the **optical power**.
  - The quantity $\gamma|A|^2$ (units of $\mathbf{m^{-1}}$) if n_2 (units of $\mathbf{m^2/W}$)
- The parameter A_{eff} (**effective mode area**) is defined as

$$A_{eff} = \frac{\left(\iint_{-\infty}^{\infty} |F(x,y)|^2 \, dxdy\right)^2}{\iint_{-\infty}^{\infty} |F(x,y)|^4 \, dxdy}. \qquad (2.3.30)$$

  - Its **evaluation** requires the use of **modal distribution *F*(*x*,*y*)** for the fundamental fiber mode.
  - A_{eff} depends on **fiber parameters** such as the **core radius** and **the core–cladding index difference**.
  - If ***F*(*x*,*y*)** is approximated by a **Gaussian distribution** , $A_{eff} = \pi w^2$.

- The **width parameter** *w* depends on the ***V*** parameter of fiber and can be obtained from **Figure 2.1** or **Eq. (2.2.10)**.
  - $\boldsymbol{A_{\text{eff}}}$ can vary in the range of **1** to **100** $\boldsymbol{\mu}$**m**^{2} in the **1.5-**$\boldsymbol{\mu}$**m** region (depending on the fiber design)
  - γ takes values in the range **1–100 W**$^{-1}$**/km (**if $\boldsymbol{n_2 \approx 2.6 \times 10^{-20}}$ **m**^{2}**/W**).
- For **highly nonlinear fibers**, $\boldsymbol{A_{\text{eff}}}$ is reduced intentionally to enhance the nonlinear effects.

$$V = p_c a = k_0 a (n_1^2 - n_c^2)^{1/2}, \qquad (2.2.10)$$

- **Eq. (2.3.28)** describes propagation of **picosecond optical pulse** in single-mode fibers

$$\frac{\partial A}{\partial z}+\beta_1\frac{\partial A}{\partial t}+\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2}+\frac{\alpha}{2}A=ir(\omega_0)|A|^2A, \qquad (2.3.28)$$

  – It is related to the **nonlinear Schrödinger (NLS) equation** and it can be reduced to that form **under certain conditions**.

  – It includes the effects

    - of **fiber losses** through $\boldsymbol{\alpha}$,
    - of **chromatic dispersion** through $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$,
    - of **fiber nonlinearity** through $\boldsymbol{\gamma}$.

- Briefly, the **pulse envelope** moves at the **group velocity** $v_g \equiv 1/\beta_1$, while the effects of **group-velocity dispersion (GVD)** are governed by β_2.
- The **GVD** parameter β_2 can be **positive** or **negative** depending on whether the wavelength λ is below or above the **zero dispersion wavelength** λ_D of the fiber
- In the **anomalous-dispersion regime** ($\lambda > \lambda_D$), β_2 is negative, and the fiber can support **optical solitons**.
- In **standard silica fibers** (the change in sign occurring in the vicinity of **1.3** μ**m**)
  - $\beta_2 \sim 50$ **ps²/km** (visible region)
  - $\beta_2 \sim -20$ **ps²/km** (near **1.5** μ**m**)
- The term on the right side of **Eq. (2.3.28)** governs the nonlinear effects of **self-phase modulation (SPM)**.

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2} A = ir(\omega_0)|A|^2 A, \qquad (2.3.28)$$

Nonlinear Shordinger Equqtion

- https://www.youtube.com/watch?v=5vw0Csy_1rs