

## 2. The Generation of Ultrashort Laser Pulses



The importance of bandwidth

More than just a light bulb

Laser modes and mode-locking

Making shorter and shorter pulses

- Pulse-pumping

- Q-switching and distributed-feedback lasers

- Passive mode-locking and the saturable absorber

- Kerr-lensing and Ti:Sapphire

- Active mode-locking

- Other mode-locking techniques

Limiting factors

Commercial lasers

**But a light bulb is also broadband.**

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What else is required to make an ultrashort pulse?

Answer: **“Mode-locking”**

Okay, what are modes and what does it mean to lock them?

# Mode locking

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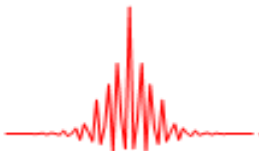
- Mode locking is a **dynamic steady-state process**, differs previous the three pulse-generation
- **Pulsed laser** action is attained by **coupling together** the modes of a laser and **locking phase** to each other
- A laser can oscillate on many **longitudinal modes**, with **frequencies** that are equally separated by the **Fabry-Perot** intermodal spacing  $\nu_F = c/2d$
- Although these modes normally **oscillate independently** (**free-running modes**), external means can be used to **couple** them and lock their phase together
- The modes can be regarded as the components of a **Fourier-series expansion** of a periodic function of time of period  $T_F = 1/\nu_F = 2d/c$ , which constitute a **periodic pulse train**.

- Each of laser modes is approximated by a **uniform plane wave** propagating with a velocity  **$c = c_0/n$**

$$U(z, t) = \sum_q A_q \exp \left[ j2\pi\nu_q \left( t - \frac{z}{c} \right) \right], \quad (15.4-25)$$

$$\nu_q = \nu_0 + q\nu_F, \quad q = 0, \pm 1, \pm 2, \dots \quad (15.4-26)$$

- For **convenience**, we assume that the  **$q = 0$**  mode **coincides with the central frequency  $\nu_0$**  of the **atomic lineshape**
- Since the **modes interact with** different groups of atoms in an **inhomogeneously broadened medium**, their **phases  $\arg\{A_q\}$**  are **random** and statistically **independent**



- Substituting (15.4-26) into (15.4-25) =>

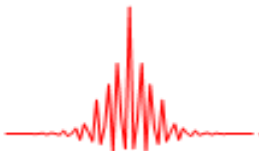
$$U(z, t) = \mathcal{A} \left( t - \frac{z}{c} \right) \exp \left[ j 2 \pi \nu_0 \left( t - \frac{z}{c} \right) \right], \quad (15.4-27)$$

- Complex envelope

$$\mathcal{A}(t) = \sum_q A_q \exp \left( \frac{j q 2 \pi t}{T_F} \right) \quad (15.4-28)$$

$$T_F = \frac{1}{\nu_F} = \frac{2d}{c}. \quad (15.4-29)$$

- If the **magnitudes** and **phases** of the **complex coefficients**  $A_q$  are **properly** chosen,  $\mathcal{A}(t)$  may be made to take the form of **periodic narrow pulses**



- Consider  $M$  modes ( $M = 2S+1$ ),  $A_q = A$ ,  $q = 0, \pm 1, \dots, \pm S$

$$A(t) = A \sum_{q=-S}^S \exp\left(\frac{jq2\pi t}{T_F}\right) = A \sum_{q=-S}^S x^q = A \frac{x^{S+1} - x^{-S}}{x - 1} = A \frac{x^{S+\frac{1}{2}} - x^{-S-\frac{1}{2}}}{x^{\frac{1}{2}} - x^{-\frac{1}{2}}}, \quad (15.4-30)$$

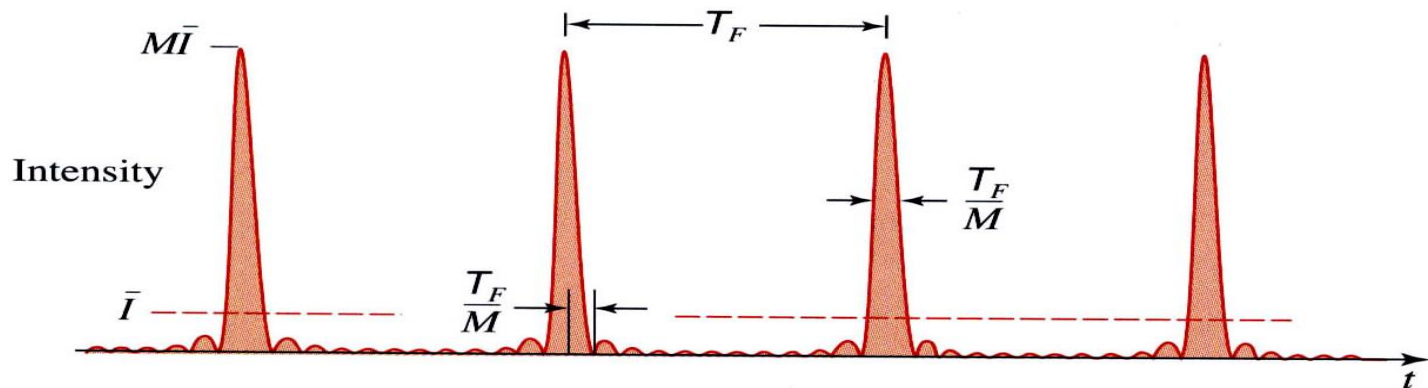


$$x = \exp(j2\pi t/T_F)$$

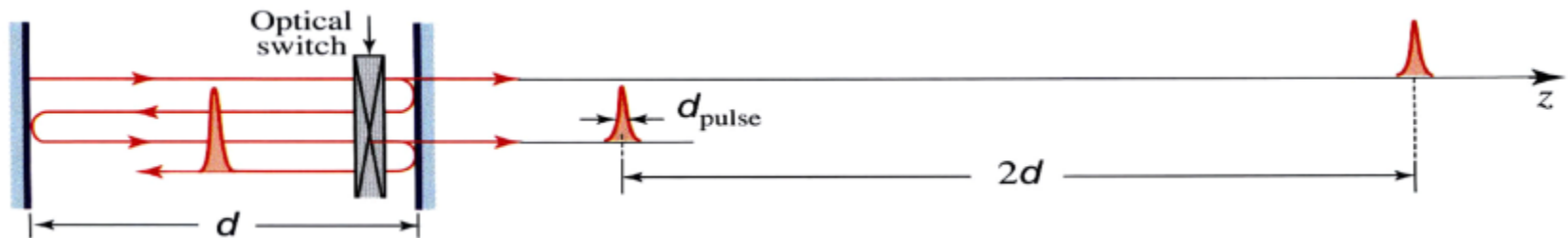
$$A(t) = A \frac{\sin(M\pi t/T_F)}{\sin(\pi t/T_F)}. \quad (15.4-31)$$

- The optical intensity =

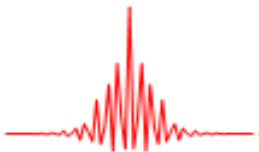
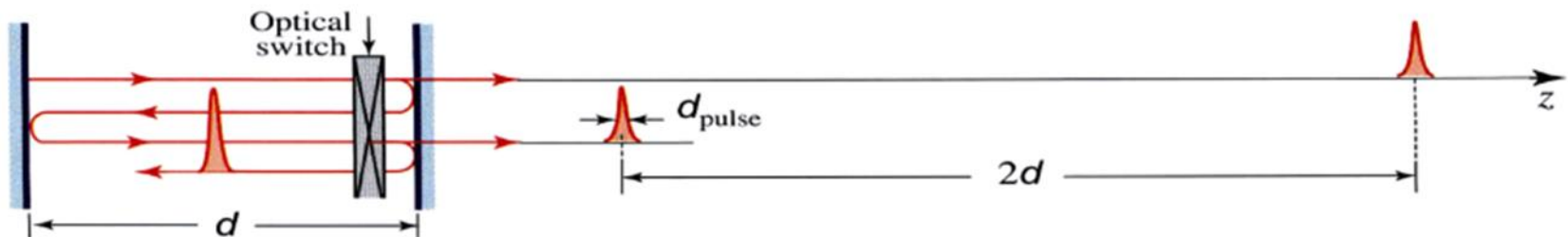
$$I(t, z) = |A|^2 \frac{\sin^2 [M\pi(t - z/c)/T_F]}{\sin^2 [\pi(t - z/c)/T_F]}, \quad (15.4-32)$$



- The shape is **dependent** on the **number of modes  $M$**
- If  $M \approx \Delta \nu / \nu_F$ ,  $\tau_{\text{pulse}} = T_F / M \approx 1 / \Delta \nu$
- Because  $\Delta \nu$  can be **quite large**, very narrow mode-locked laser pulses can be generated
- The **ratio** between the **peak** and **mean intensities** is **equal** to the number of modes  $M$  (also quite large)
- The period of the pulse train ( $T_F = 2d/c$ ) is just **the time for a single round trip of reflection** within the resonator



- The light in a **mode-locked laser** can be regarded as  
A **single narrow pulse** of photons reflecting back and forth  
between the mirrors of the resonator  
The **transmitted pulses** are separated by the **distance  $2d$**   
and have a **spatial width  $d_{\text{pulse}} = c\tau_{\text{pulse}} = 2d/M$**





➤ A particular example :  $\text{Nd}^{3+}$ :glass laser

Operating at  $\lambda_0 = 1.05 \mu\text{m}$ , refractive index  $n = 1.5$  and linewidth  $\Delta\nu = 7 \text{ THz}$

Thus , pulse duration  $\tau_{\text{pulse}} = 1/\Delta\nu \approx 140 \text{ fs}$  and pulse length  $d_{\text{pulse}} \approx 42 \mu\text{m}$

If the length of the resonator  $d = 15 \text{ cm}$ , the mode separation  $\nu_F = c/2d = 1 \text{ GHz}$

$M = \Delta\nu/\nu_F = 7000$  modes, the peak intensity is 7000 times greater than the average intensity

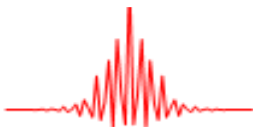
➤ In media with broad linewidths

Mode locking is generally more advantageous than Q-switching for obtaining short pulses

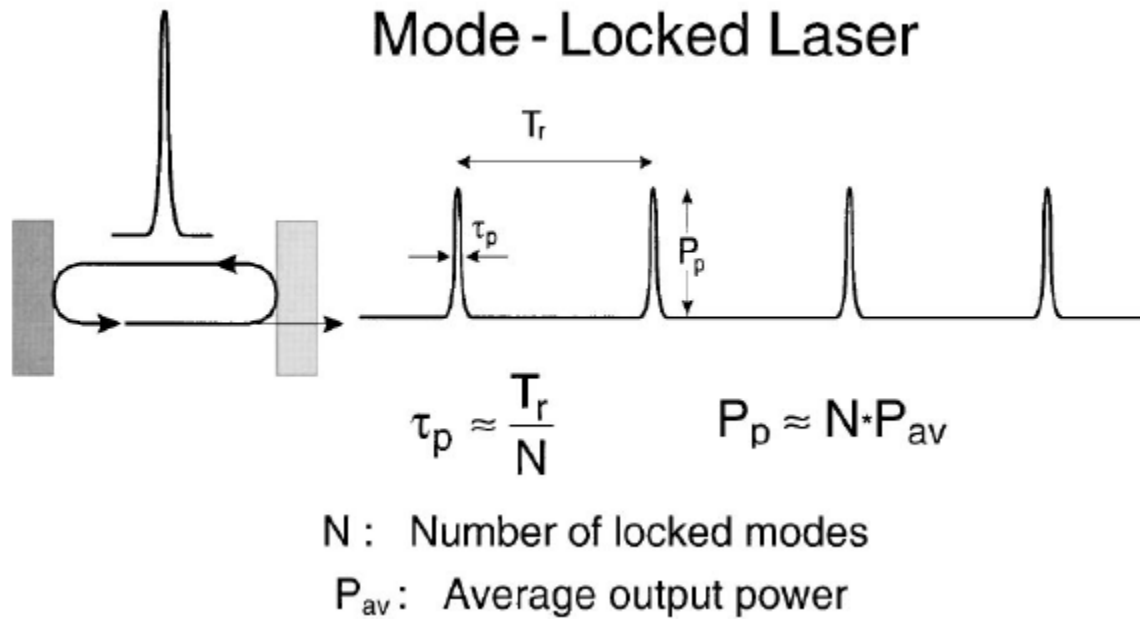
➤ Gas lasers generally have narrow atomic linewidths, so that ultrashort pulses cannot be obtained by mode locking

**Table 15.4-1** Characteristic properties of a mode-locked pulse train.

Temporal period	$T_F = \frac{2d}{c}$	Pulse duration	$\tau_{\text{pulse}} = \frac{T_F}{M} = \frac{1}{\Delta\nu}$
Spatial period	$2d$	Pulse length	$d_{\text{pulse}} = \frac{2d}{M}$
Mean intensity	$\bar{I}$	Peak intensity	$I_p = M\bar{I}$



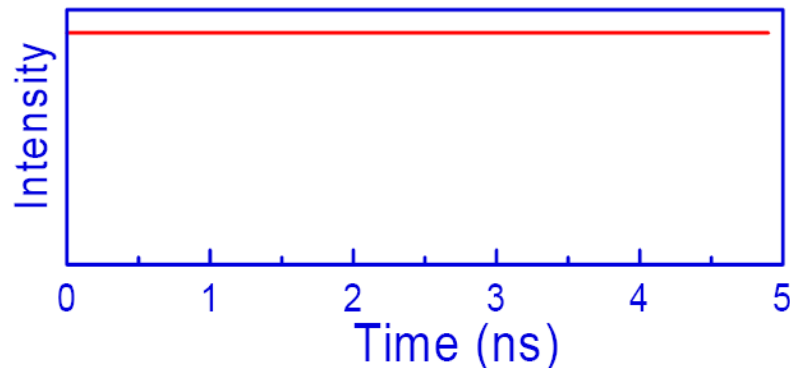
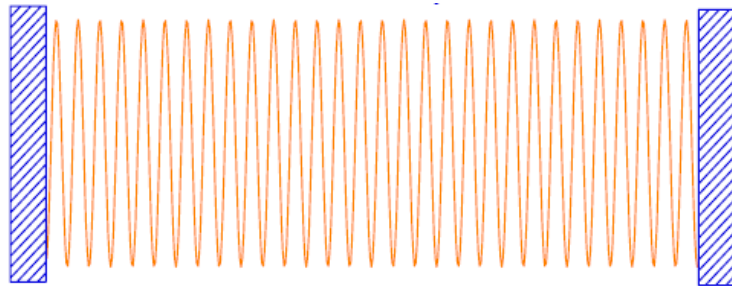
# Mode-locked laser



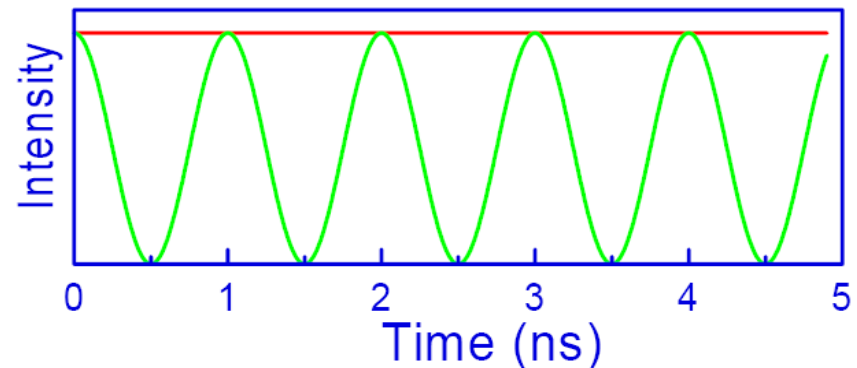
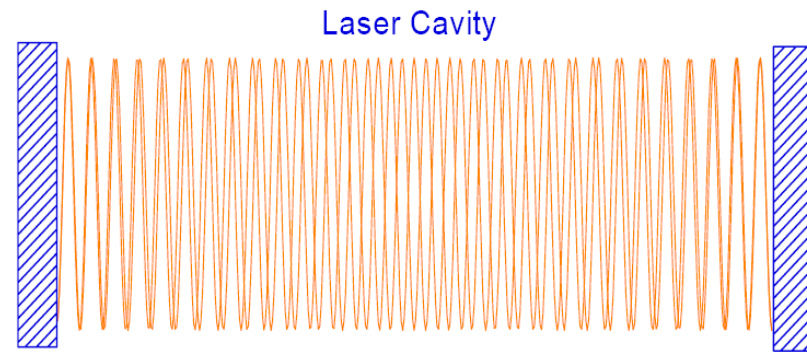
➤ Radiation power as a function of time at the output of a stationary mode-locked laser

# Mode Locking with one and two oscillation frequency

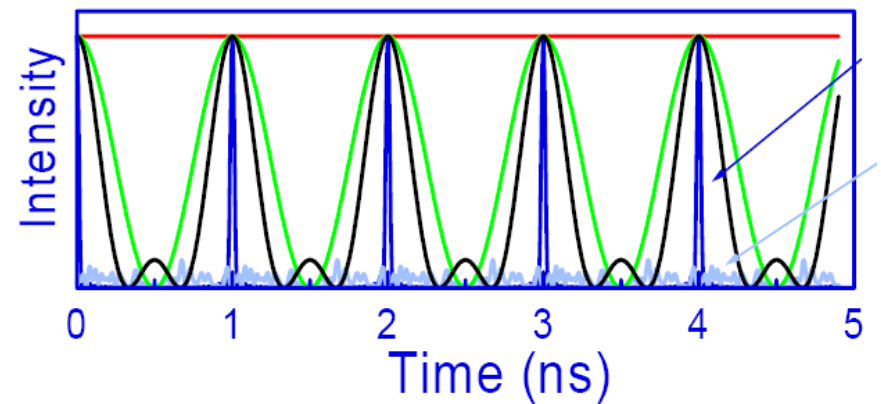
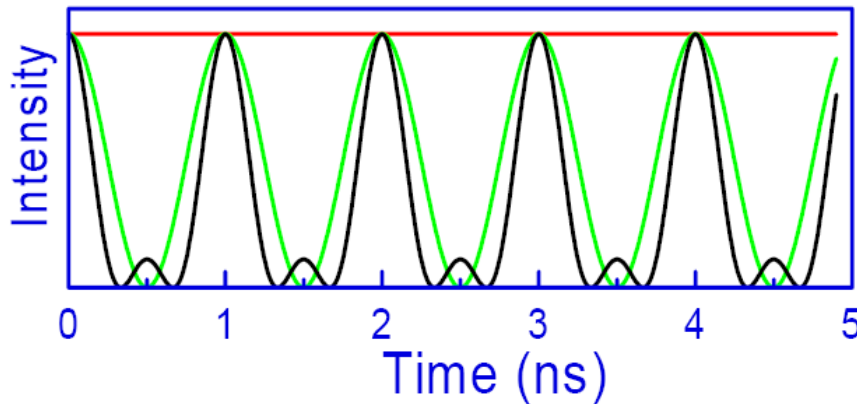
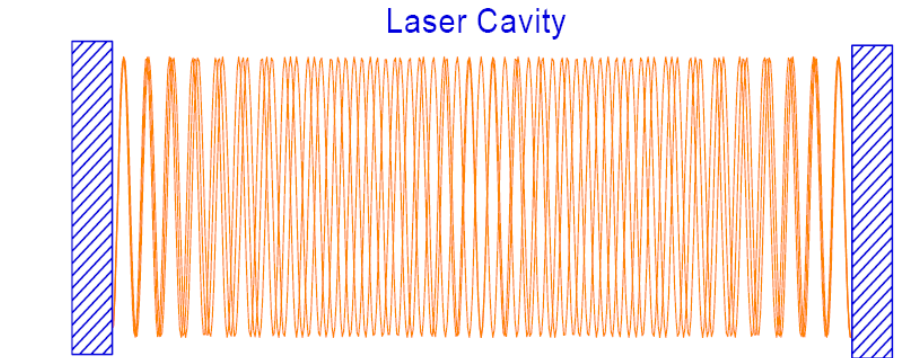
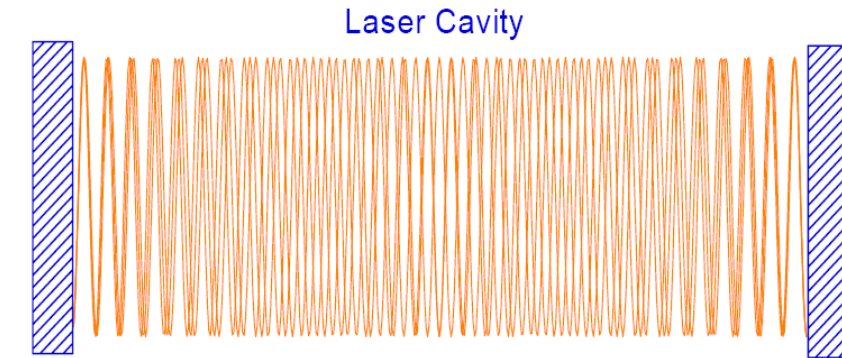
$$\vec{E}(t) = E_0 e^{i N \omega t}$$



$$\vec{E}(t) = E_0 e^{i N \omega t} + E_0 e^{i (N-1) \omega t}$$

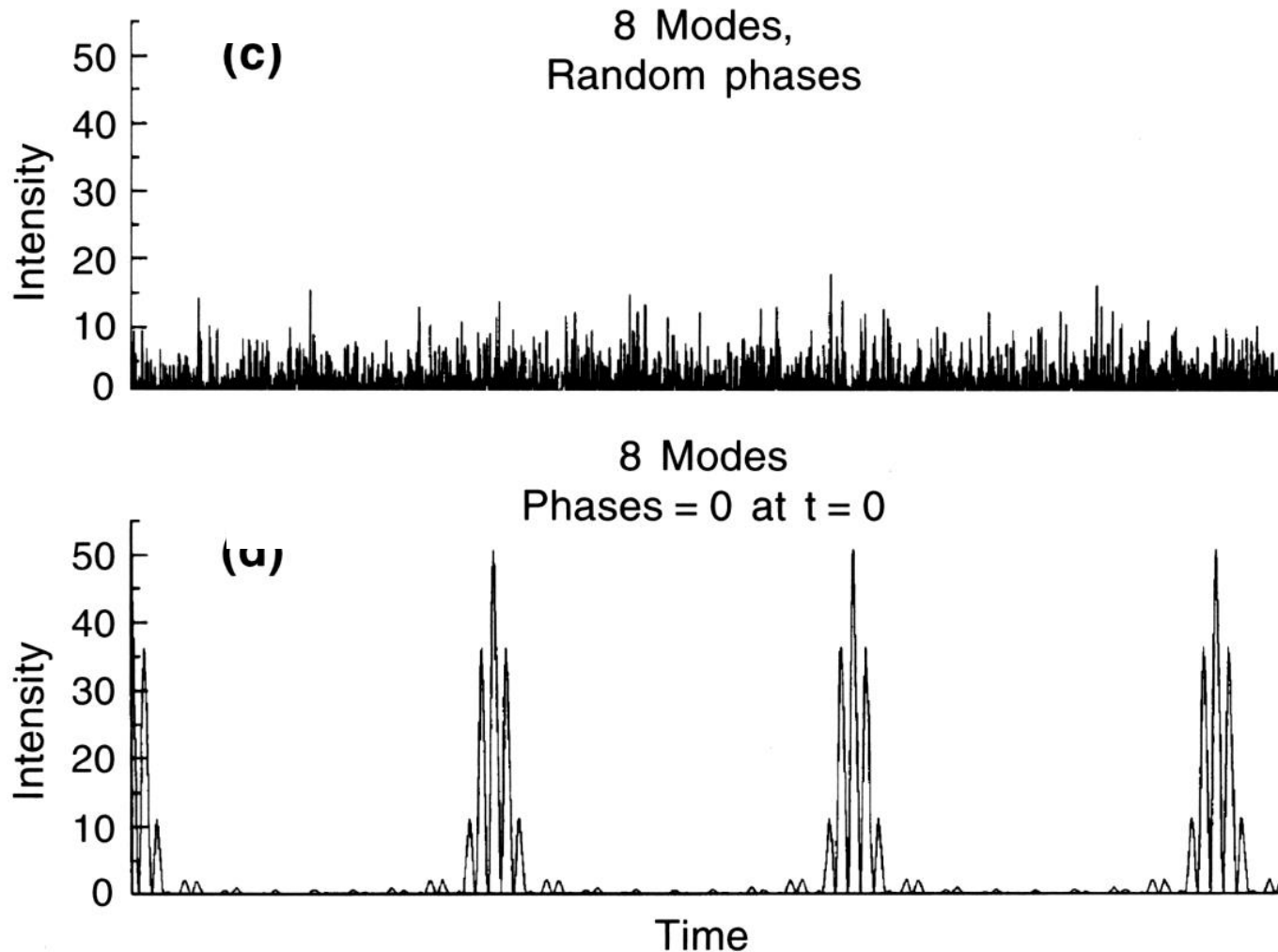


$$\vec{E}(t) = E_0 e^{i(N+1)\omega t} + E_0 e^{iN\omega t} + E_0 e^{i(N-1)\omega t}$$



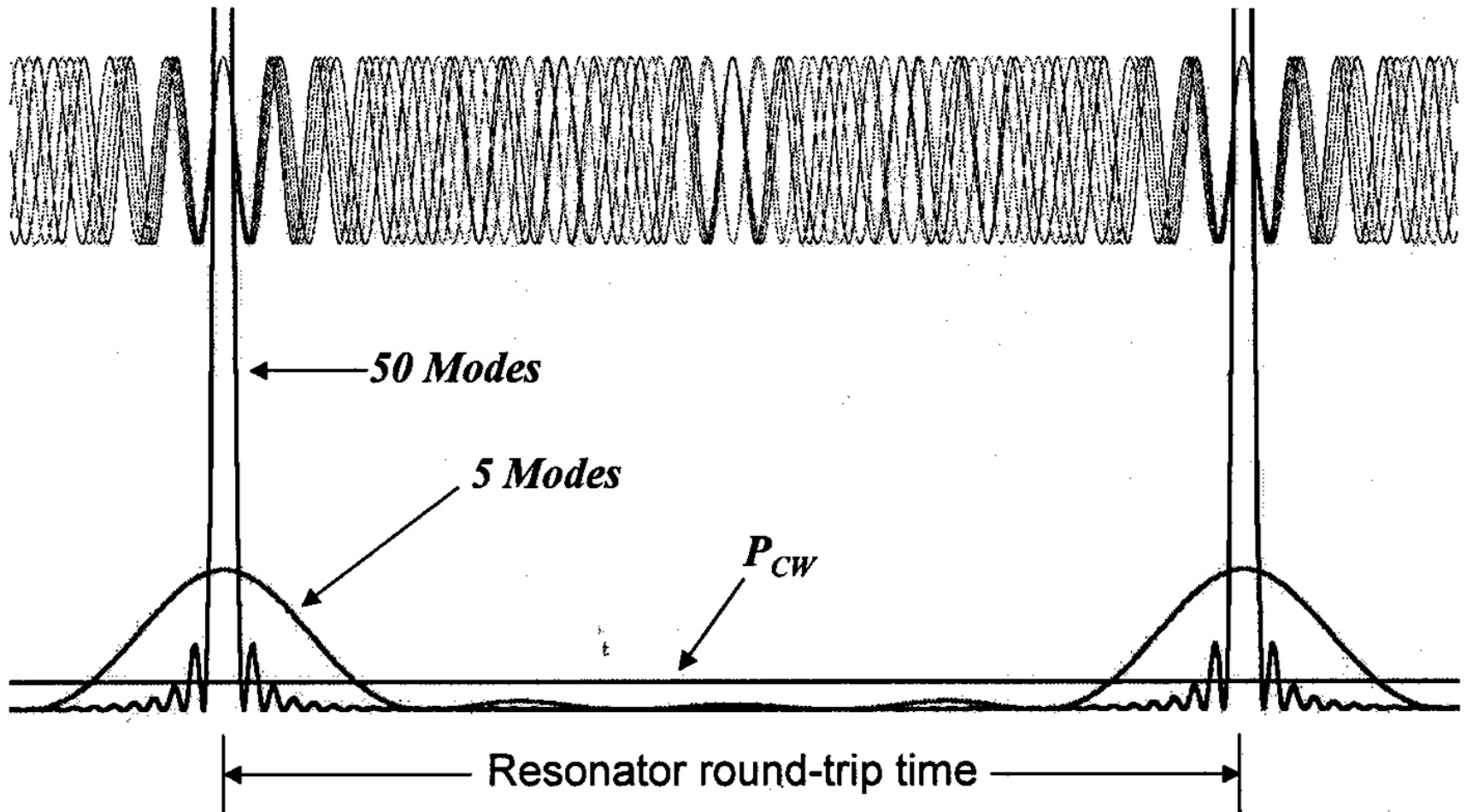
**Ultrashort pulse can be generate as number of modes M increase**

# Numerical simulation of mode-locking



**Ultrafast lasers often have thousands of modes.**

# Locked modes

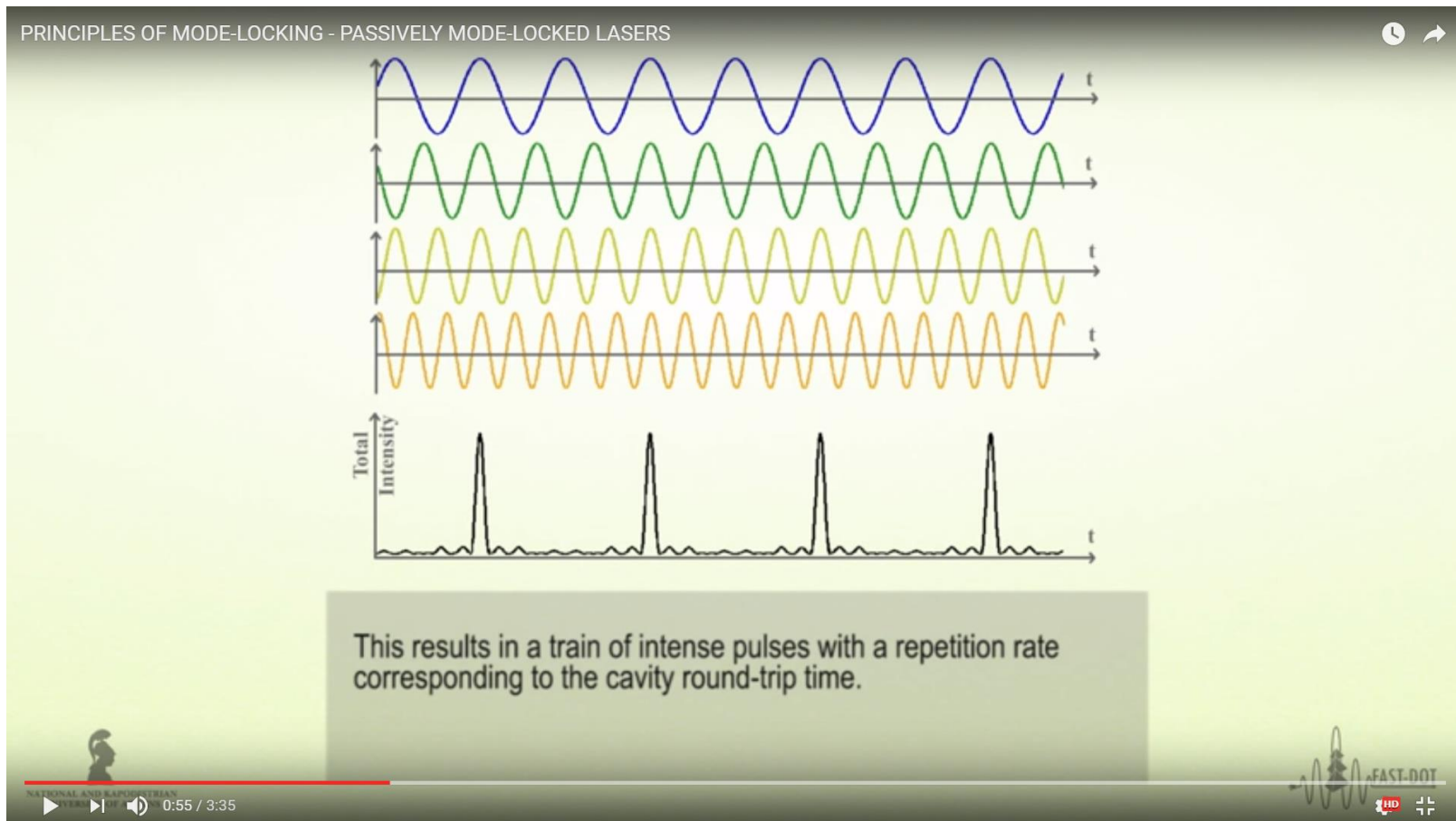


M. Didomenico, J. Appl. Phys. Lett. 35, 2870 (1964); L. Hargrove *et al.*, Appl. Phys. Lett 5, 4 (1964)

# Video of mode locking

## Linking

➤ <https://www.youtube.com/watch?v=efxFduO2Yl8>





# Lecture of mode locking

➤ <https://www.youtube.com/watch?v=Jf-hQzVsLJ8>

Lasers & Optoelectronics Lecture 23: Mode Locked Lasers (Cornell ECE4300 Fall 2016)

按下 [Esc] 即可結束全螢幕模式

The chalkboard contains the following content:

- A diagram of a laser cavity with mirrors and a gain medium. It shows the optical path length  $2L$  and the round-trip time  $2\tau$ . The frequency of the  $n$ -th mode is labeled  $\nu_n$ .
- The equation  $\omega_n = 2\pi\nu_n$ .
- A diagram showing the electric field components  $E_n(t)$  and the phase term  $e^{j(\omega_n t + n\omega_c)t}$ .
- A diagram showing the relationship between the mode number  $n$  and the frequency  $\nu_n$ .

31:14 / 50:47



# How to generate ultrafast laser

- Mode locking method:

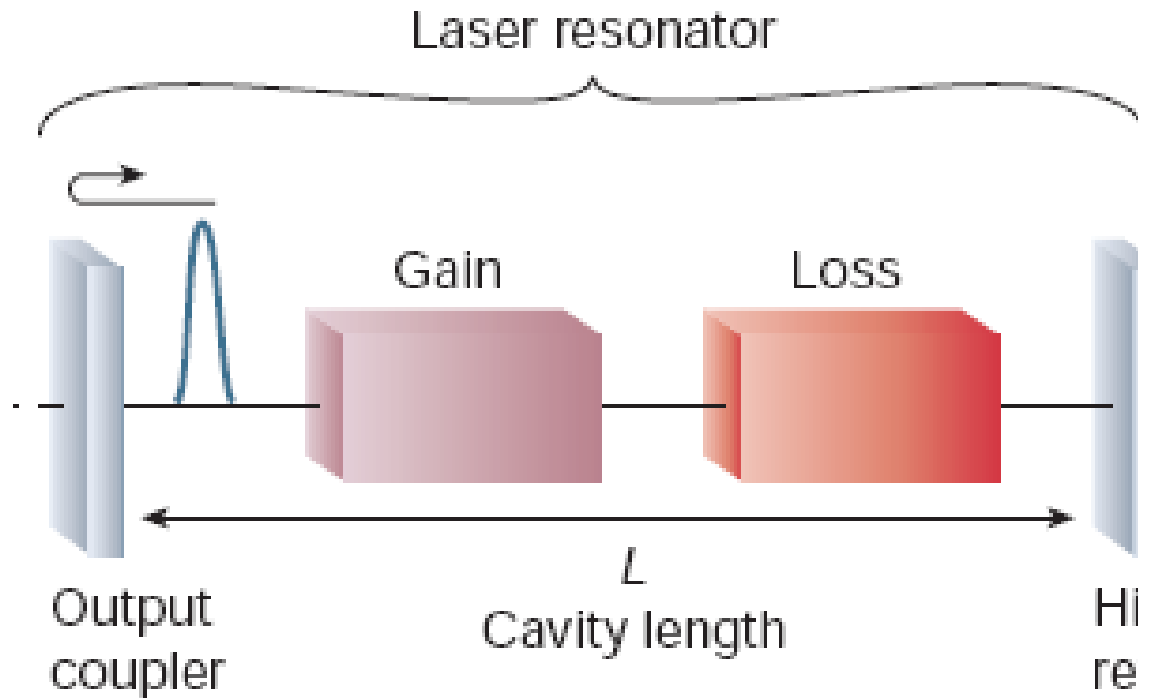
- Active mode locking

- ❑ Electro-optical modulator
- ❑ Acoustic-optical modulator

- Passive mode locking

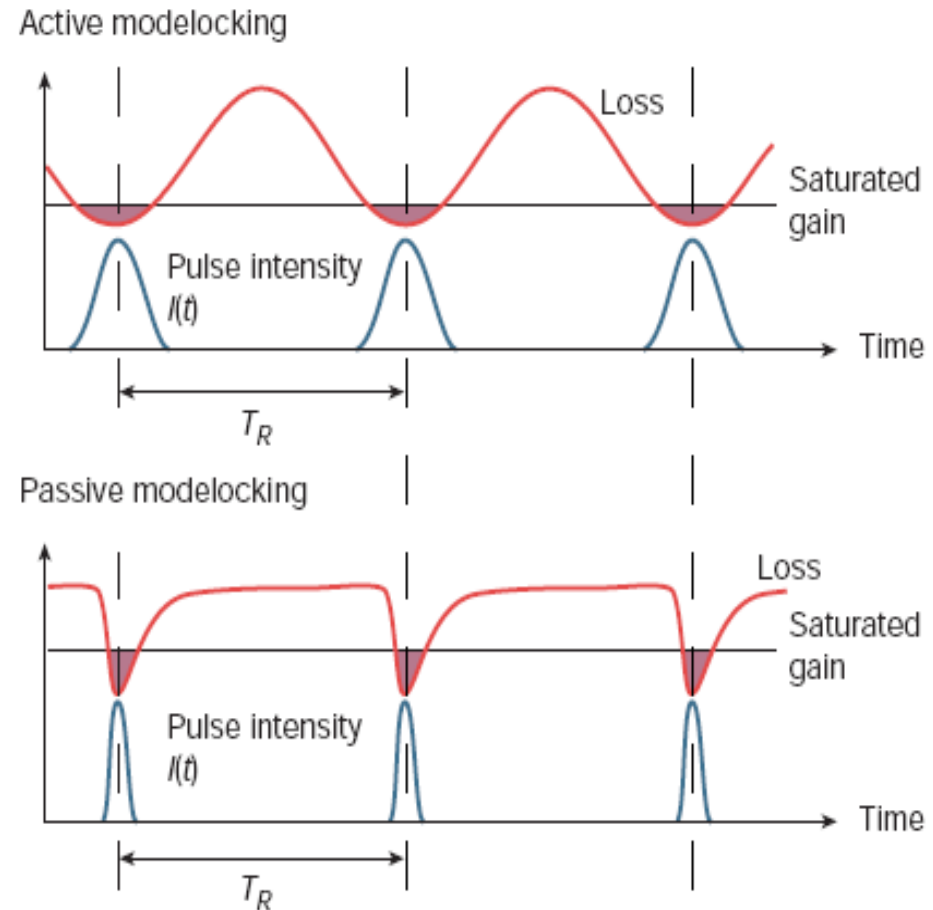
- ❑ Saturable absorber
- ❑ Semiconductor saturable absorber mirror (SESAM)
- ❑ Kerr lens mode locking
- ❑ Nonlinear mirror mode locking

A generic ultrafast laser has a broadband gain medium, a loss modulator device, and two or more mirrors:



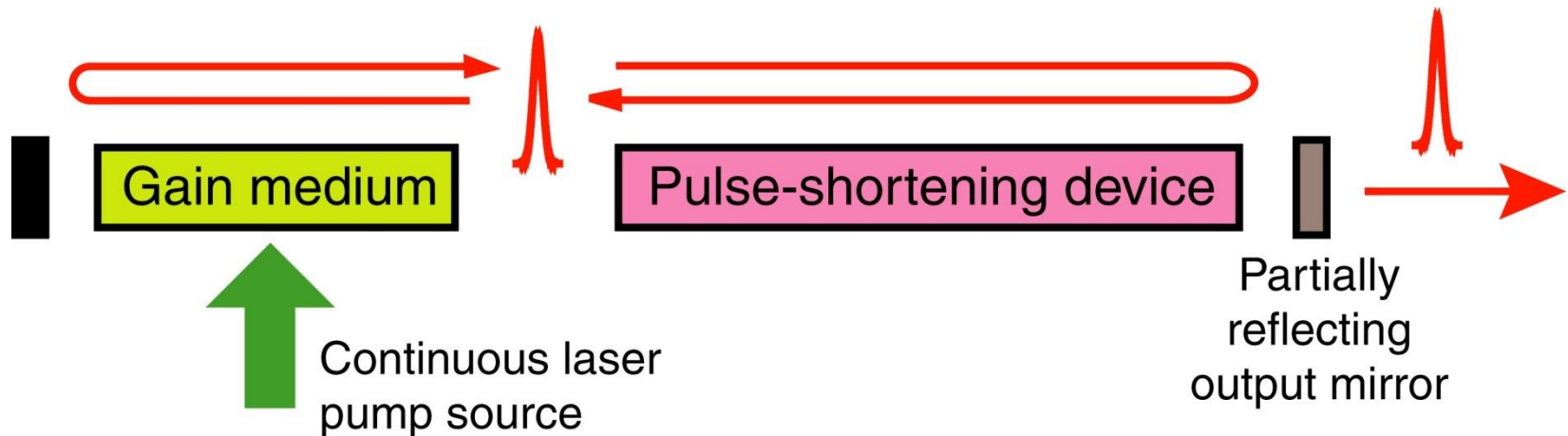
# Active and Passive Mode Locking

- The **acoustic-optical** or **electro-optical** modulator  $\Rightarrow$  periodic **sinusoidal loss modulation**  $\Rightarrow$  equal the cavity round trip time
- A **saturable absorber**  $\Rightarrow$  to obtain a self-amplitude modulation of the light inside the laser cavity.
- **Loss modulation**  $\Rightarrow$  Relatively large for low intensities but significantly smaller for a short pulse with high intensity.
- The high intensity @ the peak of pulse  $\Rightarrow$  saturates the absorber more strongly than its low intensity wings  $\Rightarrow$  **pulse shaping effect**



# A generic ultrashort-pulse laser

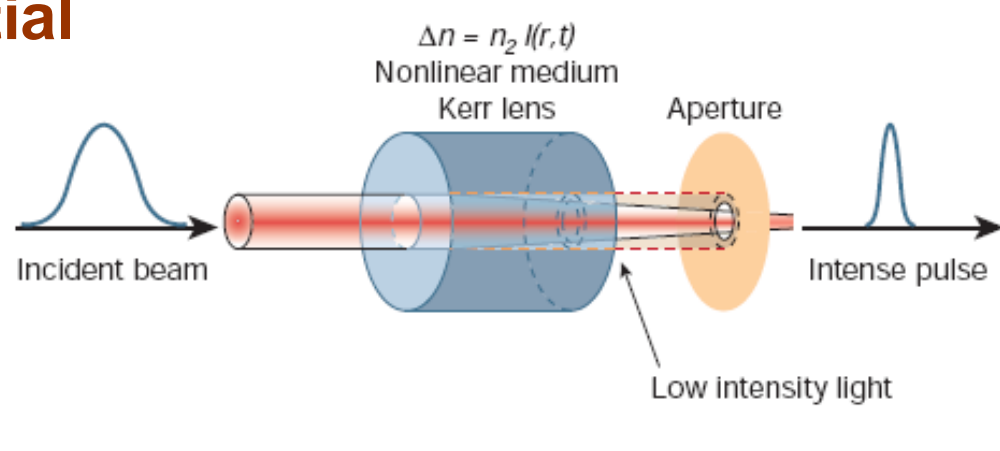
- A generic ultrafast laser has a broadband gain medium, a pulse-shortening device, and two or more mirrors:



**Many pulse-shortening devices have been proposed and used.**

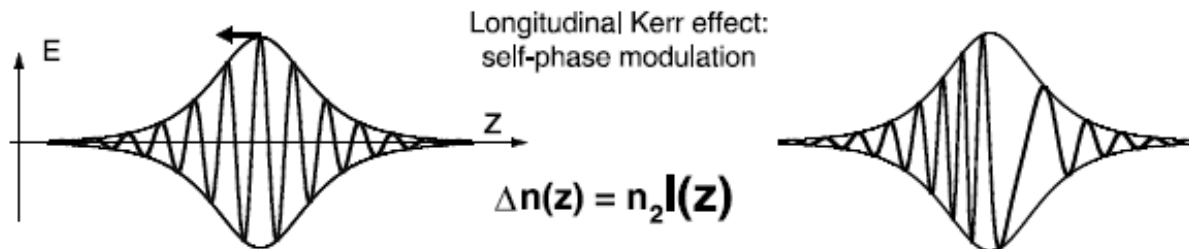
# Kerr lens mode locking

## Spatial



Self-focusing and aperture => high intensity pulse for low loss

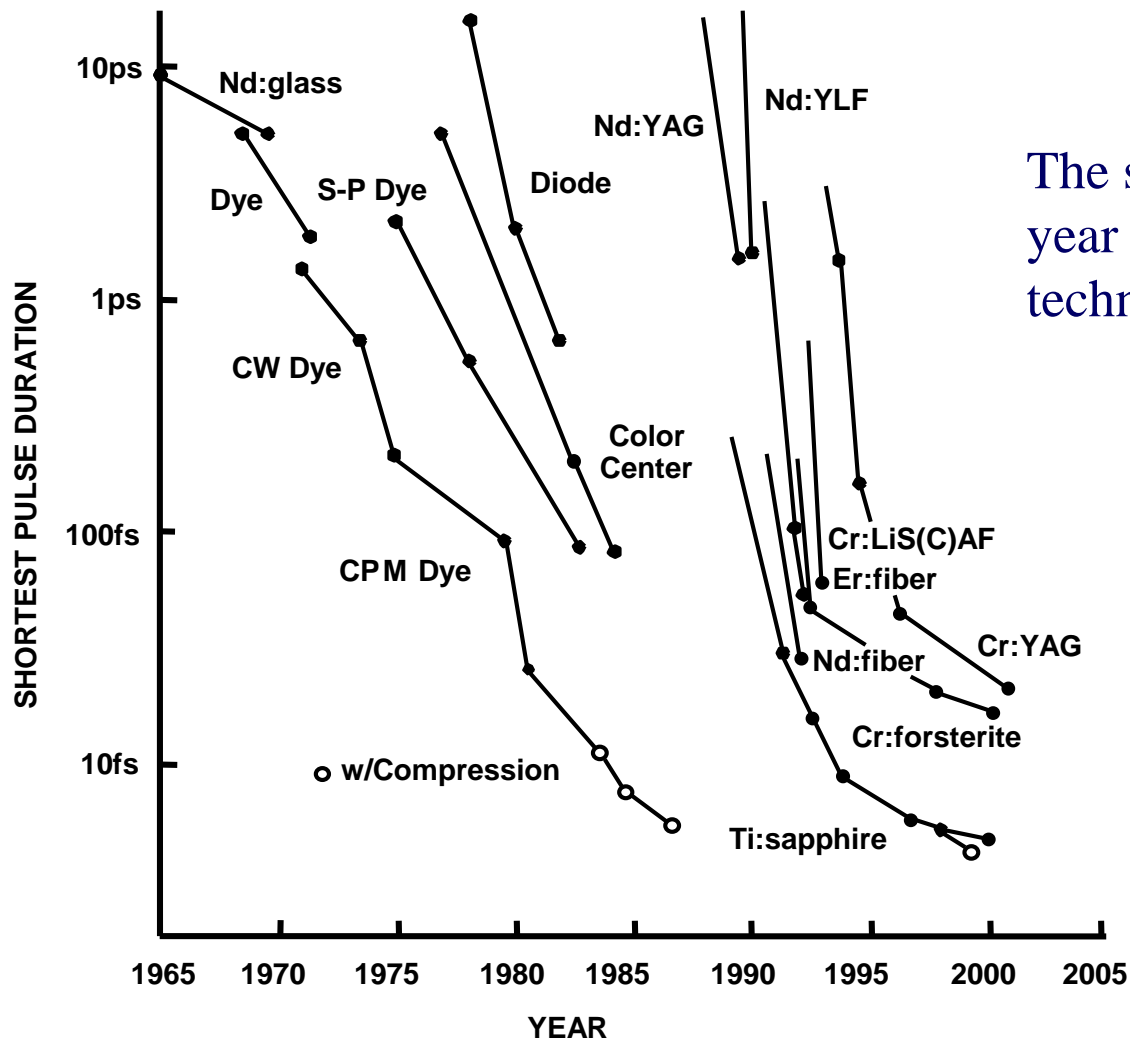
## Temporal



$\lambda$  increase @ leading edge

Self-phase modulation (SPM) and negative group velocity dispersion (GVD) => pulse shaping effect

# But first: the progress has been amazing!



The shortest pulse vs.  
year (for different  
technologies)

Erich Ippen,  
MIT

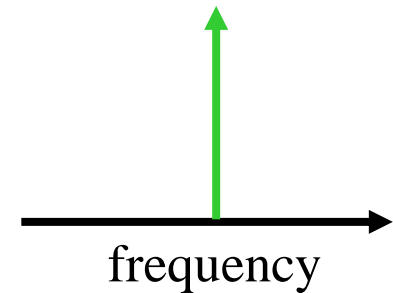
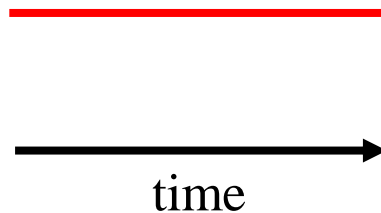
# Continuous vs. ultrashort pulses of light

- A constant and a delta-function are a Fourier-Transform pair.

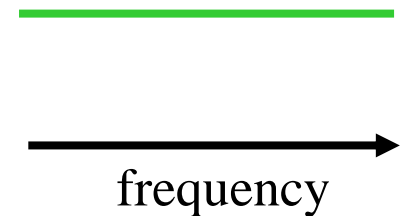
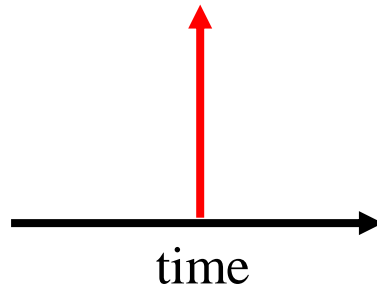
Irradiance vs. time

Spectrum

Continuous beam:

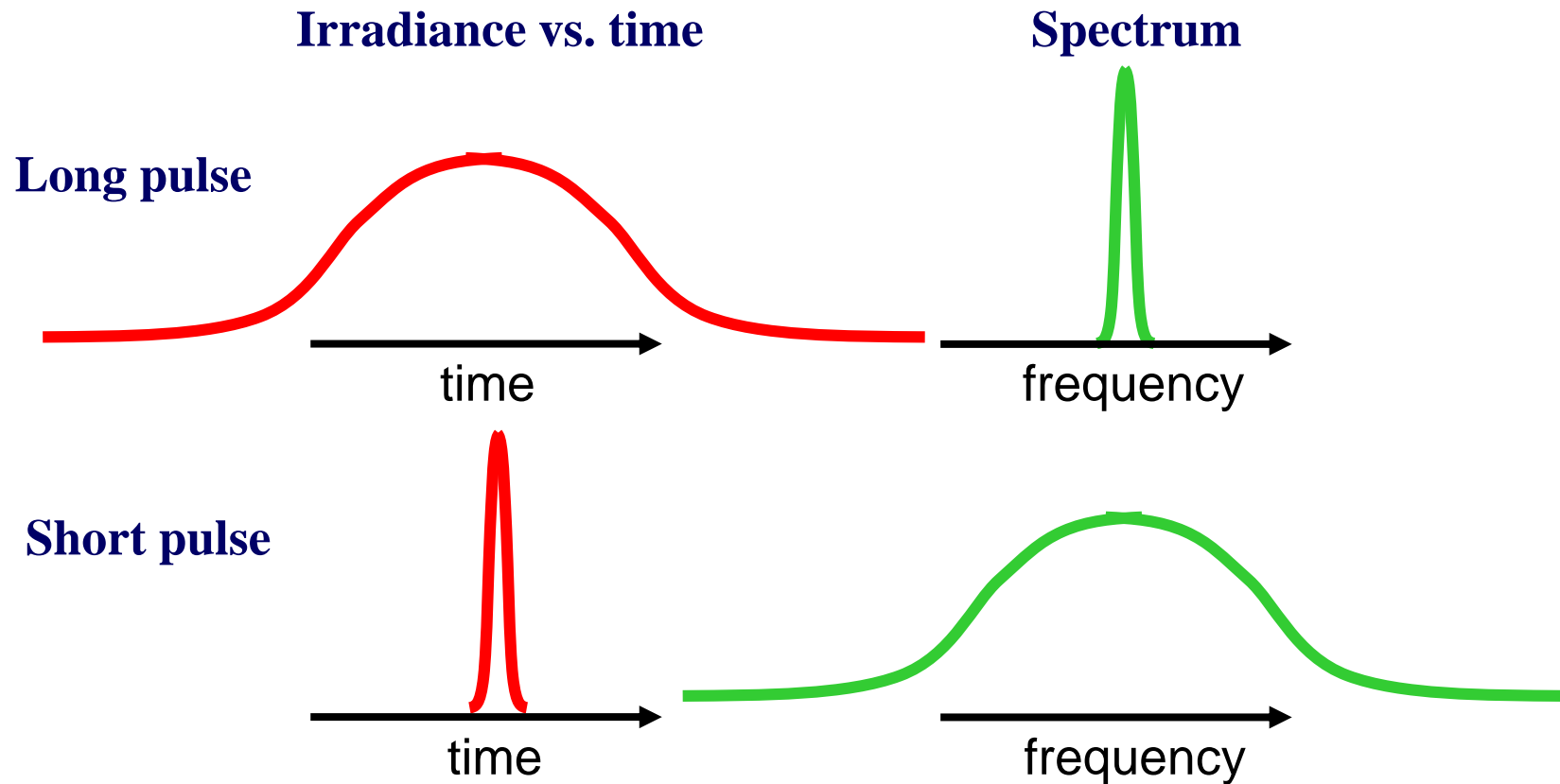


Ultrashort pulse:

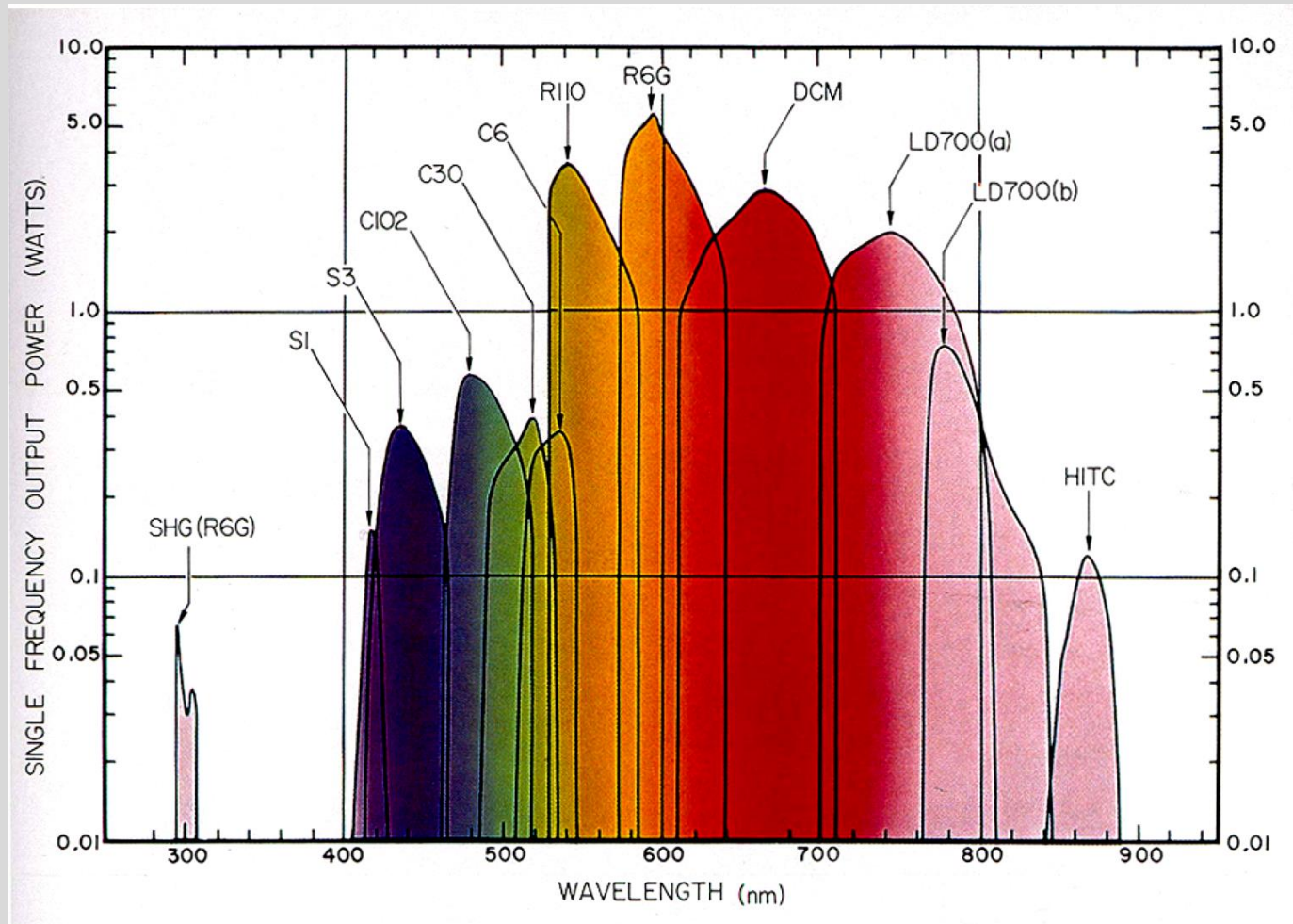


# Long vs. short pulses of light

- The uncertainty principle says that the product of the temporal and spectral pulse widths is greater than  $\sim 1$ .



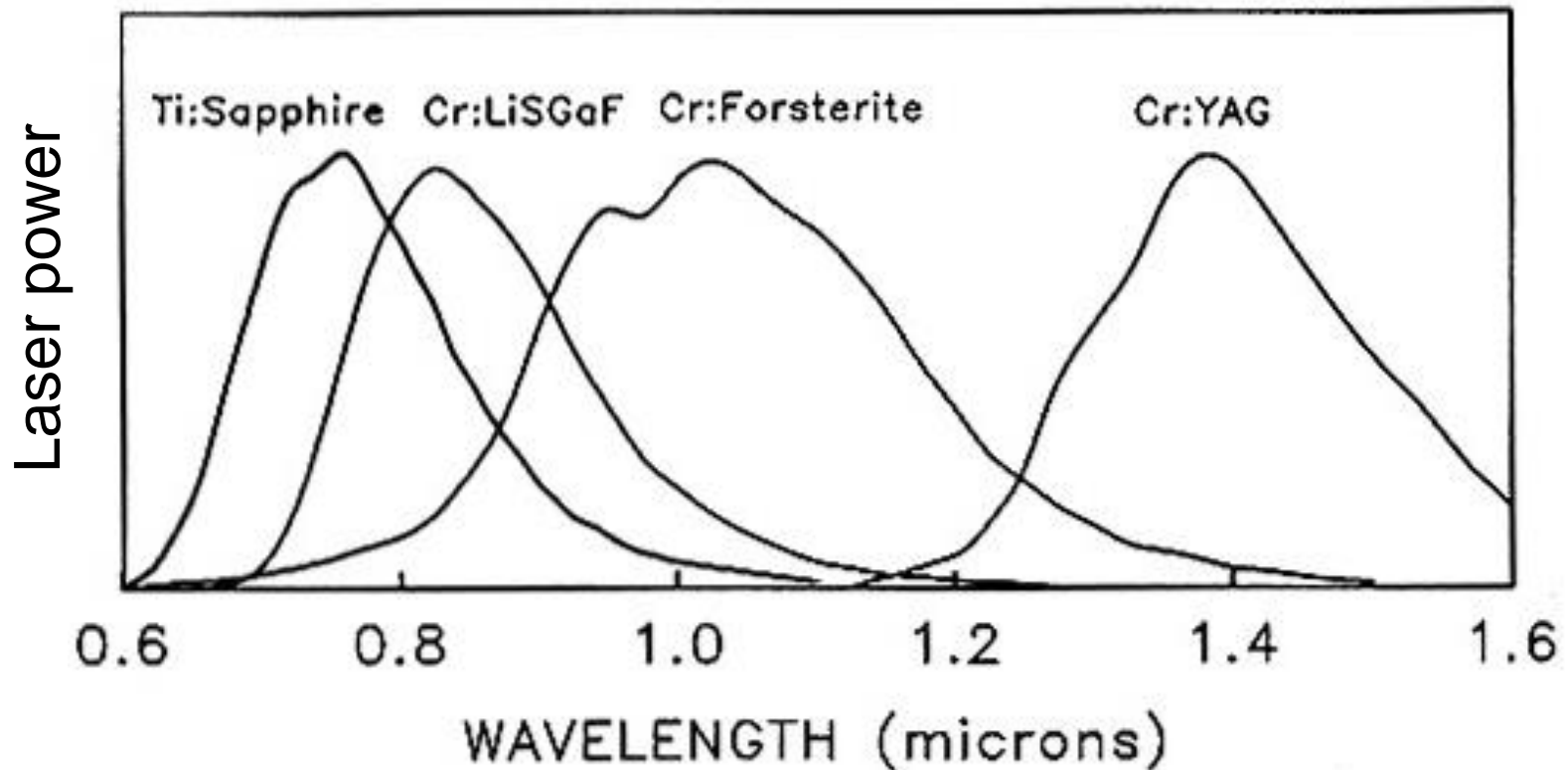
**For many years, dyes have been the broadband media that have generated ultrashort laser pulses.**





# Ultrafast solid-state laser media have recently replaced dyes in most labs.

- Solid-state laser media have broad bandwidths and are convenient.

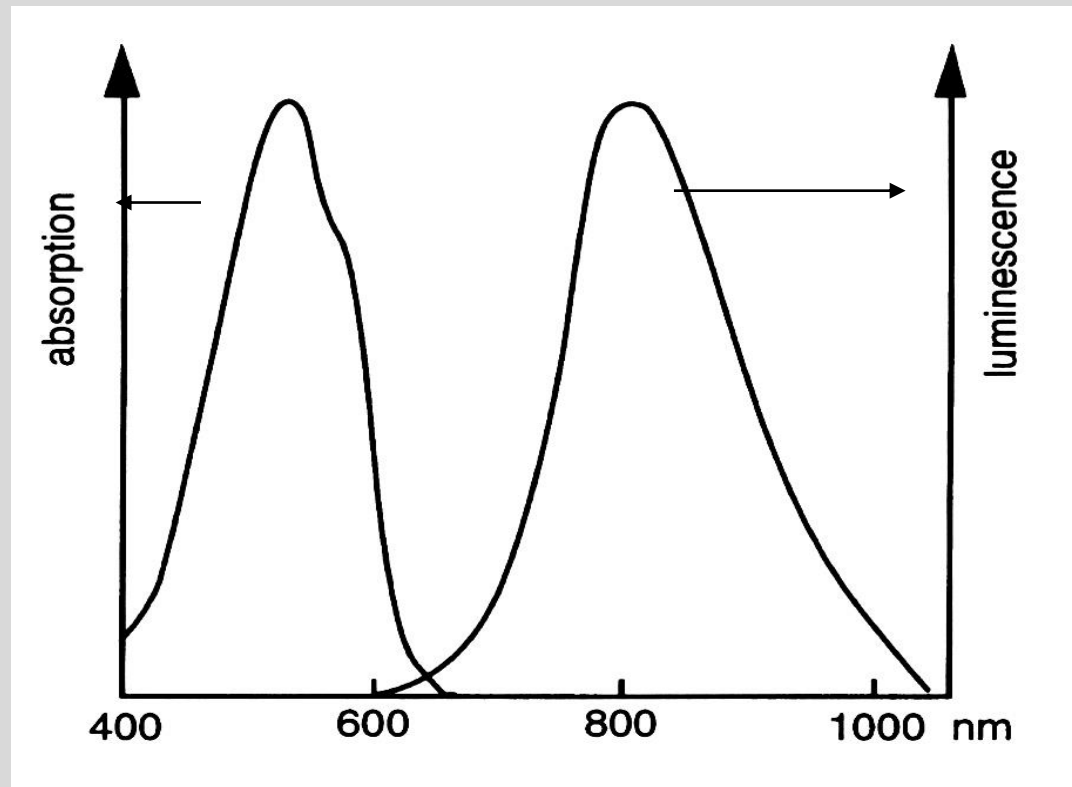


# Titanium Sapphire

➤ Ti:Sapphire is currently the workhorse laser of the ultrafast community, emitting pulses as short as a few fs and average power in excess of a Watt.

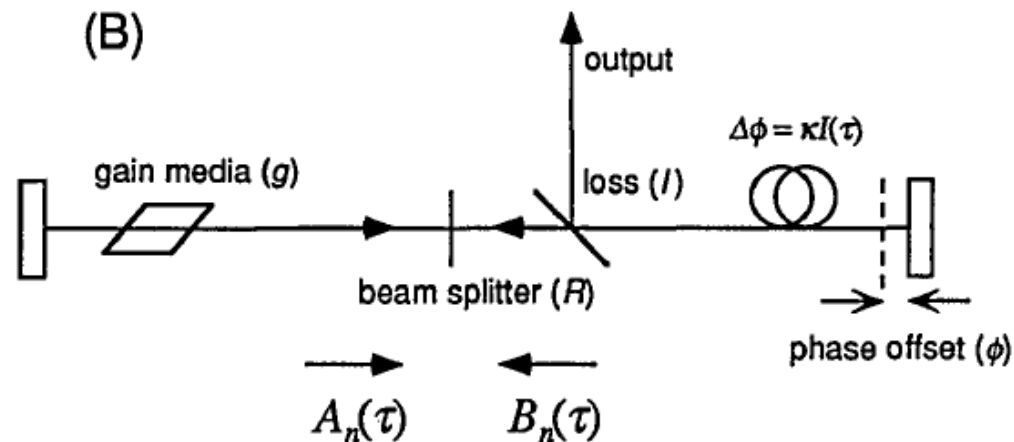
It can be pumped with a (continuous) Argon laser (~450-515 nm) or a doubled-Nd:YAG laser (532 nm).

It lases well between 700 and 1000 nm.



# Ti:sapphire laser

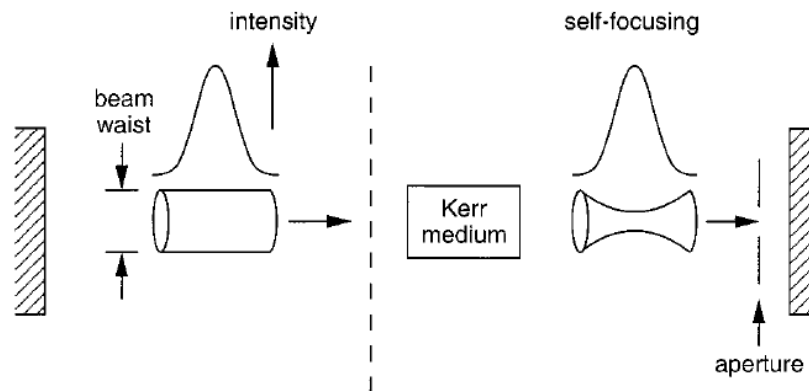
- Series of **broad-bandwidth, solid-state laser** materials were developed
  - The most notable : **Ti:sapphire ( $\text{Ti:Al}_2\text{O}_3$ )**
    - Lased over a **continuous band** stretching 680 ~ 1100 nm.
- Pulse width as shorter as **200 fs** in late 1989 using **additive pulse mode-locking (APM)**
  - APM is accomplished by **feeding back** into the laser part of its output after it has been **nonlinearly modulated** in an **external cavity**



An intracavity interferometer with a nonlinear phase modulator, to which an additive-pulse mode-locking laser is equivalent. OPTICS LETTERS / Vol. 16, No. 14 / July 15, 1991

# Ti:sapphire laser

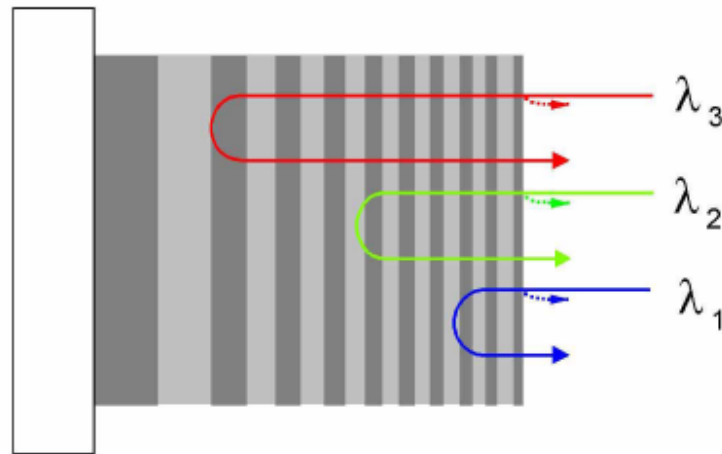
- In 1991, observed **self-mode-locking** or **Kerr lens ML**
  - Because of **nonlinearity** present in laser crystal
- Nonlinear refraction index  $n_2$  introduces an intensity dependent index given by  $n = n_0 + n_2 I$ 
  - $n_0$  : linear index of the crystal
  - $I$  : instantaneous laser pulse intensity
- **Self-focusing** when light focuses into crystal
  - Nonlinear phase delay of the beam will be highest at center of the beam
  - Additional lens in cavity with intense pulse (not for low-intensity)
- Cavity alignment can be adjusted
  - Pulse spatial mode suffers less loss than CW spatial mode



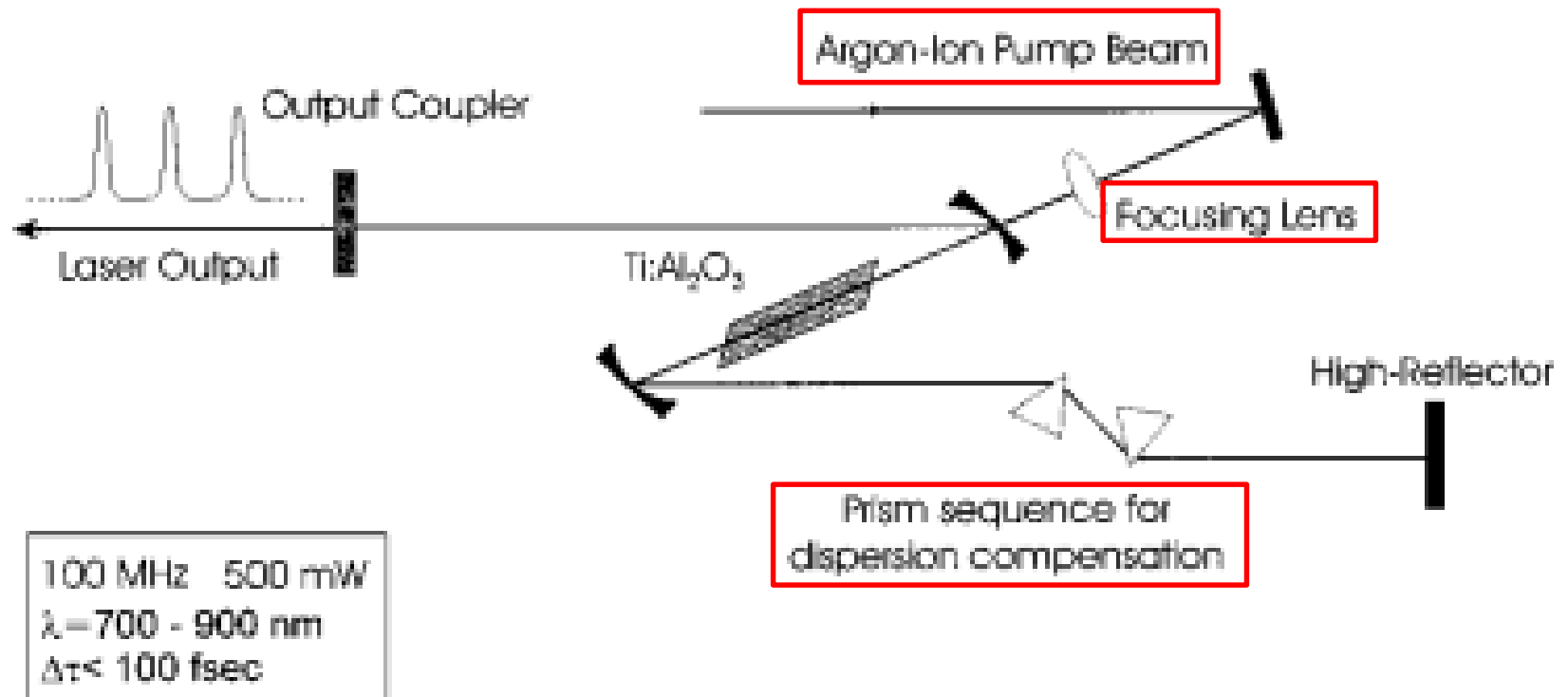
# Ti:sapphire laser

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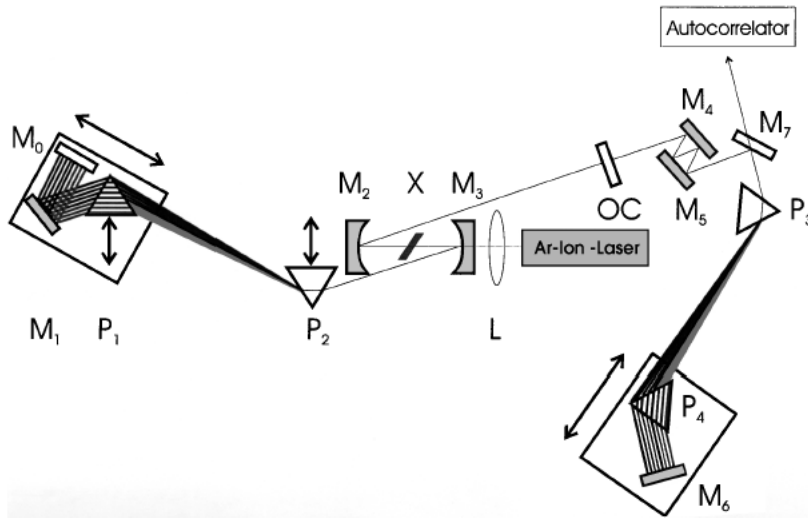
- When **self-focusing** is induced the ML profile will match the **pump mode**, and will favor pulse operation
  - Because this ML is induced by the pulse itself, is said to self-ML, and the effect is the same as if a fast SA were present in cavity
- Today Ti:sapphire lasers can generate 25 fs pulses with not to much difficulty
  - Allowing researchers to explore **semiconductor dynamics** at very short time scales
- 6.5 fs pulses from a Ti:sapphire laser
  - Combination of **prism pair & double-chirped mirror** for dispersion compensation



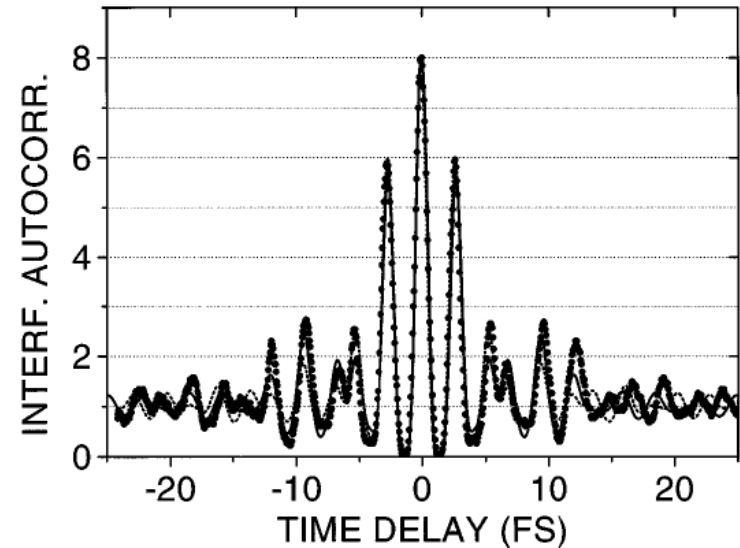
# Ti:sapphire laser (Z-configuration)



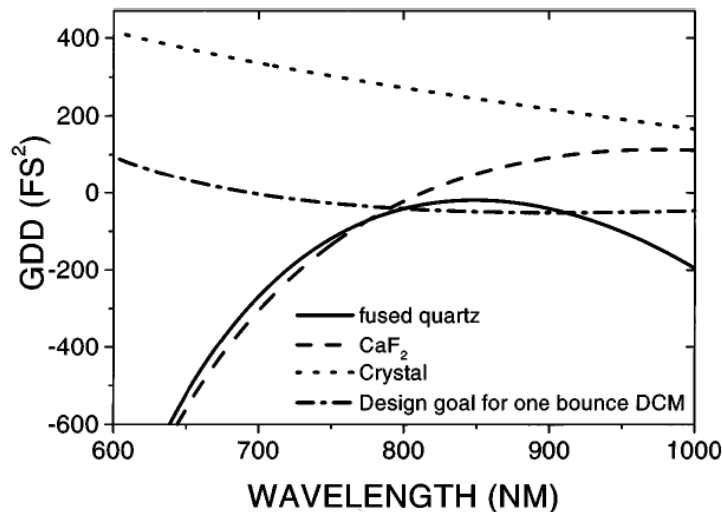
# Sub-two-cycle pulses from a Kerr-lens mode-locked Ti:sapphire laser



Interferometric autocorrelation



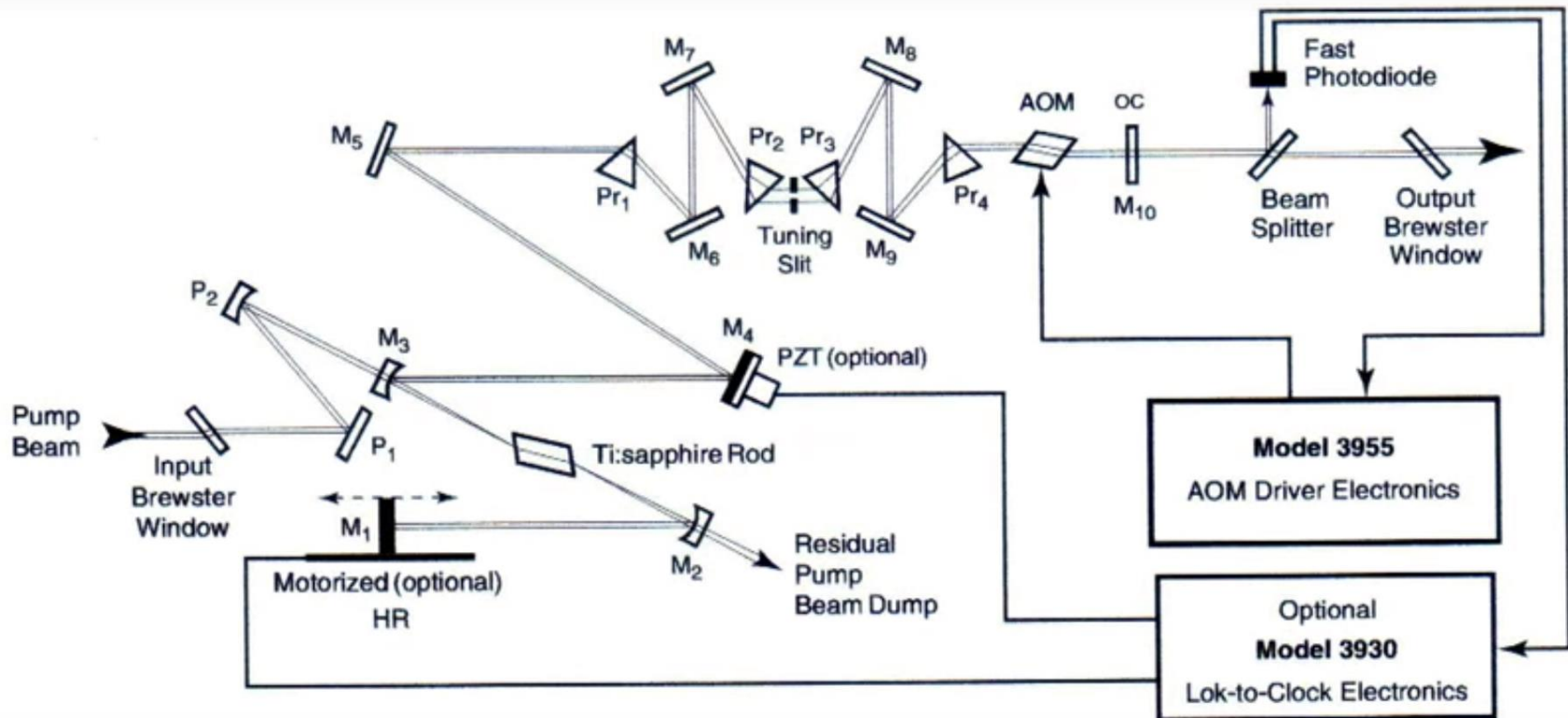
The dashed curve is a fit of a sinc function with a FWHM of 5.4 fs, and the solid curve is the calculation from the spectrum



# Video of Ti:sapphire laser

<https://www.youtube.com/watch?v=KX6h9CVXYkA>

Mode locked femtosecond titanium sapphire laser.mp4





# Pulse compression (SPM+GVD)

➤ <https://www.youtube.com/watch?v=l62aDMONhf0>

Femtosecond laser pumped by a laser diode

Compensation de la dispersion de vitesse de groupe (prismes)

The diagram illustrates a femtosecond laser system. On the left, a vertical label reads 'Ytterbium glass femtosecond laser oscillator laser diode pumped'. The main diagram shows a horizontal path of light. It starts with a narrow white pulse, followed by a yellow box labeled 'Milieu dispersif' (dispersive medium) containing a broad, multi-colored pulse. The light then enters a prism pair, represented by two prisms with a gap between them. The light path is shown as a red line that reflects off the prisms, forming a V-shape. The pulse is compressed as it exits the second prism, resulting in a narrow white pulse. A vertical label on the right reads 'SWISS-ROCKETCHMAN'. At the bottom, a video player interface shows a progress bar at 1:29 / 2:47.

# Pulse compression by GVD

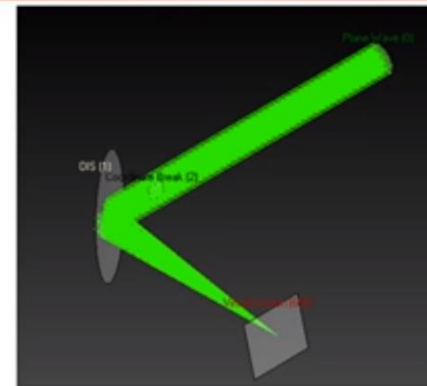
➤ <https://www.youtube.com/watch?v=BeXWyfAk5O4>

Analysis and Control of Ultrashort Pulses - An Overview

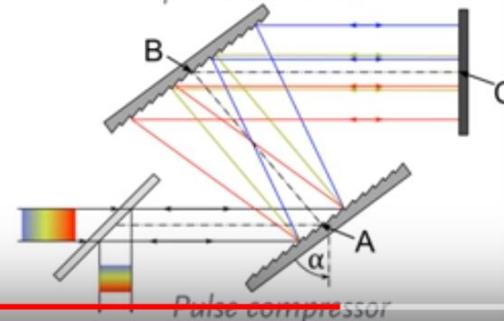
## Important optical setups for ultrashort pulses

Analysis, design and optimization of optical setups and components like:

- Focusing optics
- Pulse compressor & stretcher
- Chirped mirrors
- Refractive and diffractive beam shapers
- Gratings
- Spatial and spectral filters



*Pulse focusing by off-axis parabolic mirror*



# Signal Processing at Light Speed

➤ <https://www.youtube.com/watch?v=yh3hXI4ywq0>

Impulse Excitation of "Frequency-Independent" Antennas  
Signal Processing at Light Speed: Ultrashort Optical Pulse Generation... (Andrew Weiner)

Many antennas are highly dispersive!  
(Phase response becomes very important for time domain systems)

**Transmitter – Log-Periodic**      **Receiver – Ridged-Horn**

~1 - 2 m

**Impulse response**

**Laser generated excitation pulse**  
~20 ps

mV

Time (ps)

**~5.7 ns**

mV

Time (ns)

21. Ultrabroadband Radio-Frequency Photonics  
22. Radio-Frequency Arbitrary Waveform Generation  
23. Arbitrary Waveform Generation in the W-band  
24. Impulse Excitation of "Frequency-Independent" Antennas  
25. Shaping Wireless Waveforms that Self-Compress  
26. Waveforms That Self-Compress  
27. Spatially Selective Self-Compression  
28. Signal Processing for Quantum Optics  
29. Time-Frequency Entangled Photons (Biphotons)  
30. Orthogonal Spectral Codes

Page 1 2 3 4 5

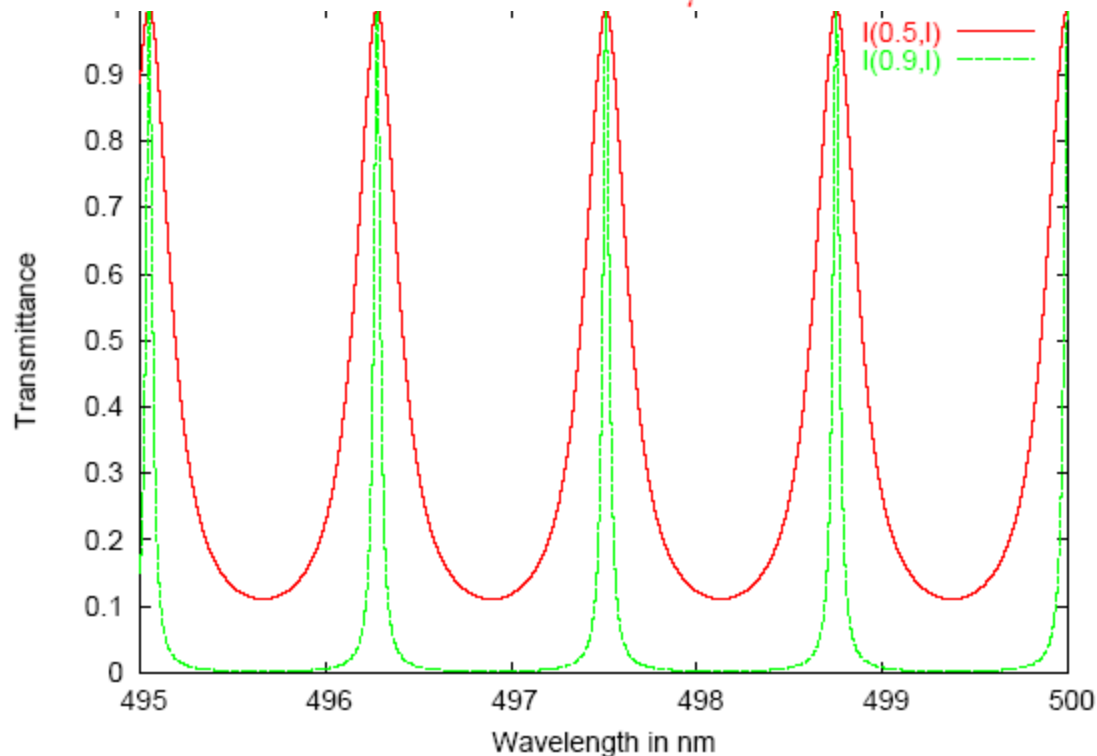
McKinney and Weiner, IEEE Trans. MTT, 54, 1681-1686 (2006); McKinney, Perouls, and Weiner, IEEE Trans. MTT, 56, 710 (2008)

48:40 / 1:10:22

PURDUE UNIVERSITY

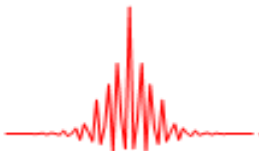
nanoBIONODE

The full output is plotted with  $d = 100\text{ }\mu\text{m}$ , for two mirrors with  $R = 0.5$  and  $R = 0.9$ .



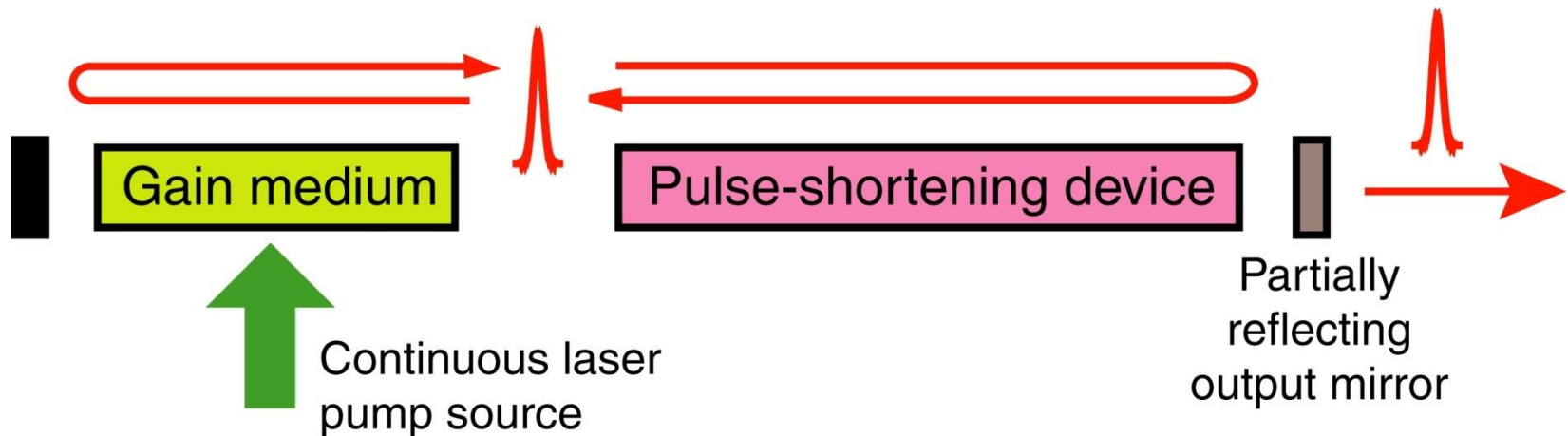
Series of transmission peaks as the reflectance increase.

The Fabry-Perot is thus operating as a very narrow band pass-filter.



# A generic ultrashort-pulse laser

- A generic ultrafast laser has a broadband gain medium, a pulse-shortening device, and two or more mirrors:



Many pulse-shortening devices have been proposed and used.

# Active and Passive Mode Locking

- The **acoustic-optical** or **electro-optical modulator**  $\Rightarrow$  periodic sinusoidal loss modulation  $\Rightarrow$  equal the **cavity round trip time**
- A saturable absorber  $\Rightarrow$  to obtain a **self-amplitude modulation** of the light inside the laser cavity.
- **Loss modulation**  $\Rightarrow$  Relatively large for **low intensities** but significantly smaller for a short pulse with high intensity.
- The **high intensity** @ the peak of pulse  $\Rightarrow$  saturates the absorber more strongly than its low intensity wings  $\Rightarrow$  **pulse shaping effect**

