超快光學

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評分方式：期中與期末報告

22. Ultrafast Optics

- 22.1 Pulse Characteristics

  A. Temporal and Spectral Characteristics

  - Temporal and Spectral Representations
  - Temporal and Spectral Widths
  - Instantaneous Frequency
  - Chirped Pulses
  - Time-Varying Spectrum

22.1 Pulse Characteristics

A. Temporal and Spectral Characteristics

- A pulse of light is described by an **optical field** of **finite time duration**.
- We represent the **field components** with **a generic normalized complex wavefunction** $U(r,t)$ such that the **optical intensity**

$$I(r,t) = |U(r,t)|^2 \quad (\mathrm{W/m^2})$$

- When we are concerned with only the **temporal** or **spectral** properties of a pulse **at a fixed position *r***
  - We will simply use the functions $U(t)$ and $I(t)$.

Temporal and Spectral Representations

- The **complex wavefunction** describing an **optical pulse** of **central frequency** ν_0 is written in

$$U(t) = A(t)e^{j\omega_0 t}$$

where

$A(t)$: complex envelope

- The **complex envelope** is characterized by its **magnitude** $|A(t)|$ and **phase** $\varphi(t) = \arg\{A(t)\}$,

- Thus, **complex wavefunction**

$$U(t) = |A(t)|e^{j[\omega_0 t+\varphi(t)]}$$

The optical intensity: $I(t) = |U(t)|^2 = |A(t)|^2$ **(W/m²)**

The energy density: $\int I(t)dt$ **(J/m²)**

Temporal and Spectral Representations

- The **intensity profiles** of **typical pulses** include :

Gaussian function : $I(t) \propto e^{-\frac{2t^2}{\tau^2}}$

Lorentzian function : $I(t) \propto \dfrac{1}{1+t^2/\tau^2}$

hyperbolic secant function $\mathrm{I(t)} \propto \sec h^2(t/\tau)$

- In the **spectral domain**, the pulse is described by the **Fourier transform**

$$V(\nu) = \int U(t) e^{-j2\pi\nu t} dt$$

Complex function: $V(\nu) = |V(\nu)|\, e^{j\psi(\nu)}$

where **$\psi(\nu)$: spectral phase**

Spectral intensity: $S(\nu) = |V(\nu)|^2$

Temporal and Spectral Representations

- The **Fourier transform** of the **complex envelope A(ν)** is **centered** at ν= 0

$$A(\nu) = \int A(t)e^{-j2\pi\nu t}dt = V(\nu - \nu_0)$$

- If the pulse has a **narrow spectral width**, then the **complex envelope** is a **slowly varying function** of time, but this is **not** the case for ultranarrow pulses with ultrawide spectral distributions.

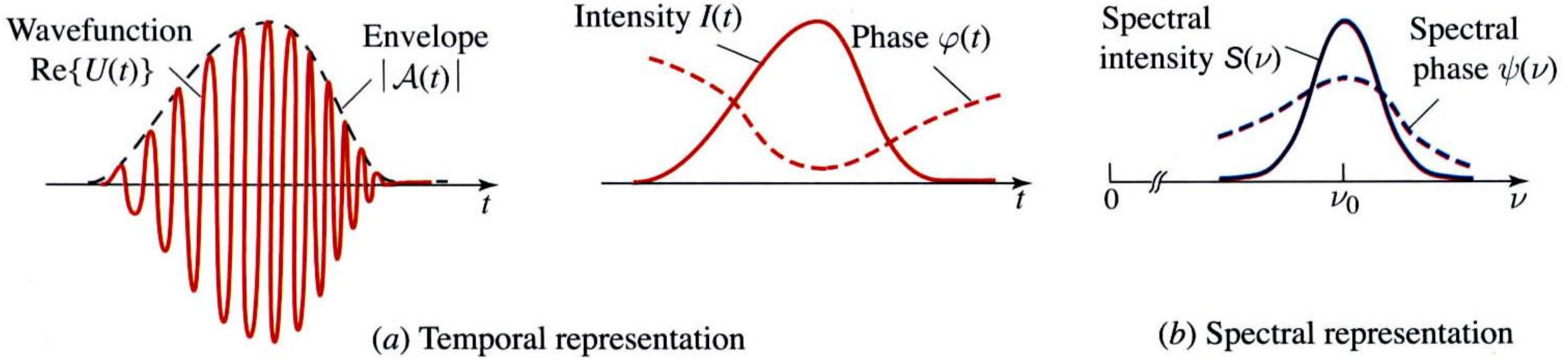


Figure 22.1-1 Temporal and spectral representations of an optical pulse. (*a*) The real part of the wavefunction $\mathrm{Re}\{U(t)\} = |\mathcal{A}(t)|\cos[\omega_0 t + \varphi(t)]$, the magnitude of the envelope $|\mathcal{A}(t)|$, the intensity $I(t)$, and the phase $\varphi(t)$. (*b*) Spectral intensity $S(\nu)$ and spectral phase $\psi(\nu)$.

Temporal and Spectral Widths

- We will use the **full-width half-max (FWHM)** definition and denote the **temporal** and **spectral widths** as τ_{FWHM} and $\Delta\nu$.

Optical intensity: $I(t) = |U(t)|^2 = |A(t)|^2$

The spectral intensity: $S(\nu) = |V(\nu)|^2$

- The **spectral width** is inversely proportional to the **temporal width** because of the **Fourier transform relation** between $U(t)$ and $V(\nu)$.

- For a **Gaussian pulse**

$$\Delta\nu = \frac{0.44}{\tau_{FWHM}}$$

22.1 Pulse Characteristics

- The **spectral intensity** $S(\nu)$ is often plotted as a function of the wavelength, $S_\lambda(\lambda)$.

$$S_\lambda(\lambda) = S(\nu)\left|\frac{d\nu}{d\lambda}\right| = \left(\frac{c}{\lambda^2}\right)S\left(\frac{c}{\lambda}\right)$$

$$\int S_\lambda(\lambda)d\lambda = \int S(\nu)d\nu$$

- For ultranarrow pulses, ($\Delta\nu << \nu_0$)

$$d\lambda = \frac{d\lambda}{d\nu}d\nu$$

$$\Delta\lambda \approx \frac{d\lambda}{d\nu}\Delta\nu$$

$$\because \lambda_0 = \frac{c}{\nu_0}$$

$$\boxed{\Delta\lambda \approx \frac{\lambda_0^2}{c}\Delta\nu,} \qquad (22.1\text{-}1)$$

Spectral Width

- For ultranarrow pulses, large $\Delta\nu$

$$\Delta\lambda = \frac{c}{\nu_0 - \frac{\Delta\nu}{2}} - \frac{c}{\nu_0 + \frac{\Delta\nu}{2}} = \frac{\lambda_0^2}{c}\frac{\Delta\nu}{1-(\frac{\Delta\nu}{2\nu_0})^2}$$

- A 2-fs pulse,
  - spectral width $\Delta\nu = 220\text{THz}$,
  - $\Delta\lambda = 847\ \text{nm}$ at $\lambda_0 = 1\,\mu\text{m}$,
  - i.e., the spectrum is quite broad and extends from visible through infrared.

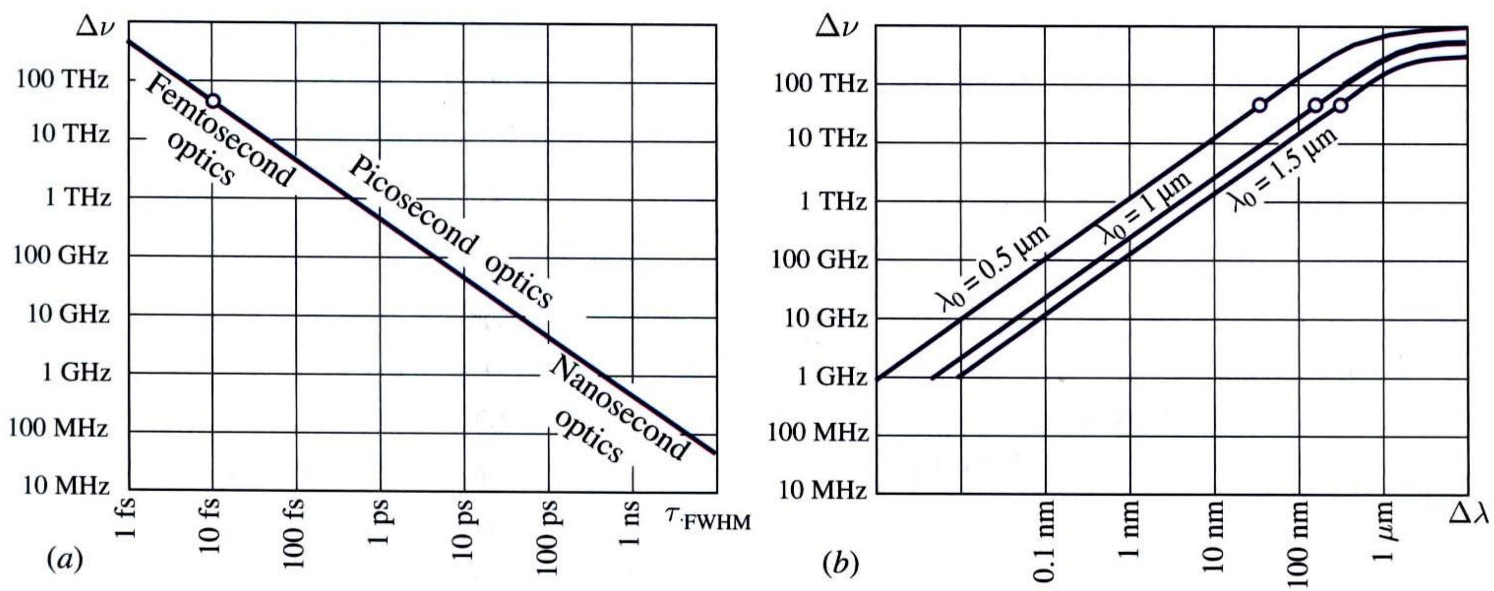


Figure 22.1-2 (a) The relation $\Delta\nu = 0.44/\tau_{\text{FWHM}}$ between the spectral width $\Delta\nu$ and the temporal width τ_{FWHM} for a Gaussian pulse. (b) The corresponding width $\Delta\lambda$ for a pulse of central frequency ν_0 corresponding to the central wavelengths $\lambda_0 = c/\nu_0 = 0.5\ \mu\text{m}$, $1\ \mu\text{m}$, and $1.5\ \mu\text{m}$. As an example, a 10-fs pulse has a spectral width $\Delta\nu = 44$ THz, corresponding to $\Delta\lambda = 37$ nm, 147 nm, and 331 nm, if the central wavelength is $\lambda_0 = 0.5\ \mu\text{m}$, $1\ \mu\text{m}$, and $1.5\ \mu\text{m}$, respectively, as indicated by the open circles in the graph. This relation is linear if $\Delta\nu \ll \nu_0$ [see (22.1-1)].

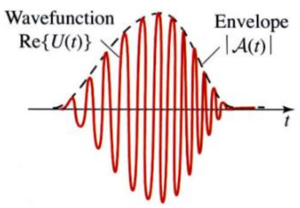
Instantaneous Frequency

- The **instantaneous angular frequency** ω_i is the **derivative** of the **phase** of $U(t)$, and the **instantaneous frequency** $\nu_i = \omega_i / 2\pi$

$$\boxed{\omega_i = \omega_0 + \frac{d\varphi}{dt}, \qquad \nu_i = \nu_0 + \frac{1}{2\pi}\frac{d\varphi}{dt}.} \qquad (22.1\text{-}4)$$

Instantaneous Frequency

- If the **phase** is a **linear function** of time, $\varphi(t) = 2\pi f t$, then the **instantaneous frequency** $\nu_i = \nu_0 + f$



- A **linearly varying phase** corresponds to a **fixed frequency shift**.
- **Nonlinear time dependence** of the **phase** corresponds to **time-dependent instantaneous frequency**.

Chirped Pulses

- A pulse is said to be **chirped**, or **frequency modulated(FM)**, if its instantaneous frequency is time varying.
- If v_i is **increasing function** of time at the pulse center (t=0), the pulse is said to be **up-chirped,** i.e., $\varphi'' = d^2\varphi/dt^2 > 0$
- If v_i is **decreasing function** of time at the pulse center, the pulse is said to be **down-chirped,** i.e., $\varphi'' = d^2\varphi/dt^2 < 0$

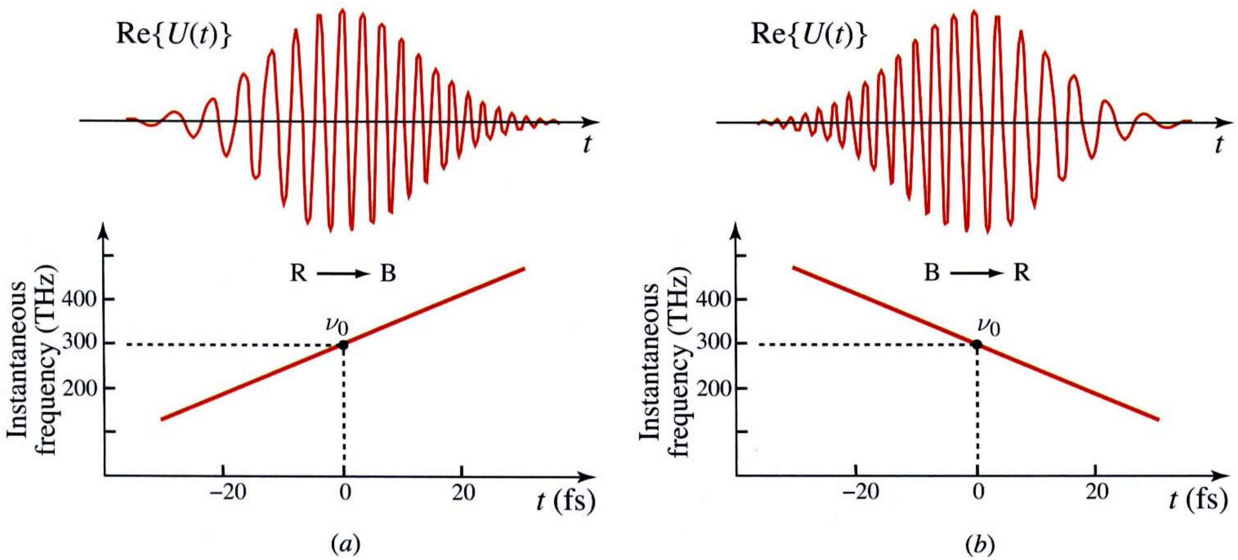


Figure 22.1-3 Linearly up-chirped and down-chirped optical pulses. (a) An up-chirped pulse has an increasing instantaneous frequency. (b) A down-chirped pulse has a decreasing instantaneous frequency. In this figure, the pulse width is 20 fs and the central frequency $\nu_0 = 300$ THz. The letters R and B, which represented red and blue, are generic indicators of long and short wavelengths, respectively.

- If the phase of an optical pulse of width τ is a **quadratic function** of time

$$\varphi(t) = \frac{at^2}{\tau^2}$$

a **: constant**

$$\Rightarrow \quad \varphi''(t) = \frac{2a}{\tau^2}$$

$$\omega_i = \omega_0 + \frac{d\varphi}{dt}, \qquad \nu_i = \nu_0 + \frac{1}{2\pi}\frac{d\varphi}{dt}. \qquad (22.1\text{-}4)$$

Instantaneous Frequency

$$\nu_i = \nu_0 + \left(\frac{a}{\pi\tau^2}\right)t$$

- The **instantaneous frequency** is a **linear function of time**.
- The pulse is said to be **linearly chirped**.

$$a = \tfrac{1}{2}\varphi''\tau^2 \qquad (22.1\text{-}5)$$

Chirp Parameter

- $a > 0$,the pulse is up-chirped.
- $a < 0$,the pulse is down-chirped.

- At $t = \tau / 2$, the **instantaneous frequency** increases by $a / 2\pi\tau$, which is of the **order of magnitude** of $a\Delta\nu$

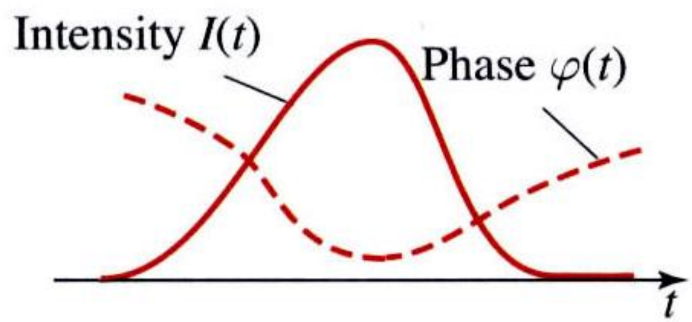
$$\nu_i = \nu_0 + (\frac{a}{\pi\tau^2})t \quad \Rightarrow \quad \nu_{\frac{\tau}{2}} = \nu_i(\frac{\tau}{2}) = \nu_0 + (\frac{a}{2\pi\tau}) \approx \nu_0 + a\Delta\nu$$

$$\delta\nu = \nu_{\frac{\tau}{2}} - \nu_0 = a\Delta\nu$$

- Thus, the **chirp parameter** is **indicative** of the **ratio** between the **instantaneous frequency change** $\delta\nu$ at the **pulse half-width point** and the **spectral width** $\Delta\nu$.

$$a \approx \frac{\delta\nu}{\Delta\nu}$$

Chirped Pulses



- If the **dependence of the phase φ on time** is an **arbitrary nonlinear function**.
- Then it can be approximated by a **Taylor-series expansion** in the **vicinity** of the **pulse center**.
- The **chirp coefficient *a*** represents the **lowest-order chirping effect** resulting from the **quadratic term** of the expansion.

Time-Varying Spectrum

- It is often useful to trace the **spectral changes** of a **time-varying pulse** throughout its **time course**.
  - Such changes are **obscured** in the **Fourier transform**, which only provides an **average spectral representation** of the entire signal **without noting** which frequencies occur at which time.
  - This is particularly evident if the **signal** is composed of a **sequence segments** each with a **different spectral composition**.
- A commonly used measure is based on a **sliding windows** (or **gate**) that selects only one **short time segment** at a time, and obtains **Fourier transform** of the pulse within the **window duration**.

Time-Varying Spectrum

- It ***W*(*t*)** is a **windows function** of **short duration** *T* beginning at *t* =0 .
- It ***U*(*t*)** is the **pulse wavefunction** .
- The product ***U*(*t*)*W*(*t*-*τ*)** is a segment of the pulse of duration ***T*** beginning at time ***τ***
- The **Fourier transform** of segment

$$\Phi(\nu,\tau) = \int U(t)W(t-\tau)\exp(-j2\pi\nu t)dt. \qquad (22.1\text{-}6)$$

Short-Time Fourier Transform

- **Spectrogram**

$$S(\nu,\tau) = \left|\Phi(\nu,\tau)\right|^2$$

Time-Varying Spectrum

Fig. 22.1-4, the result is plotted as a function of both frequency and time delay.

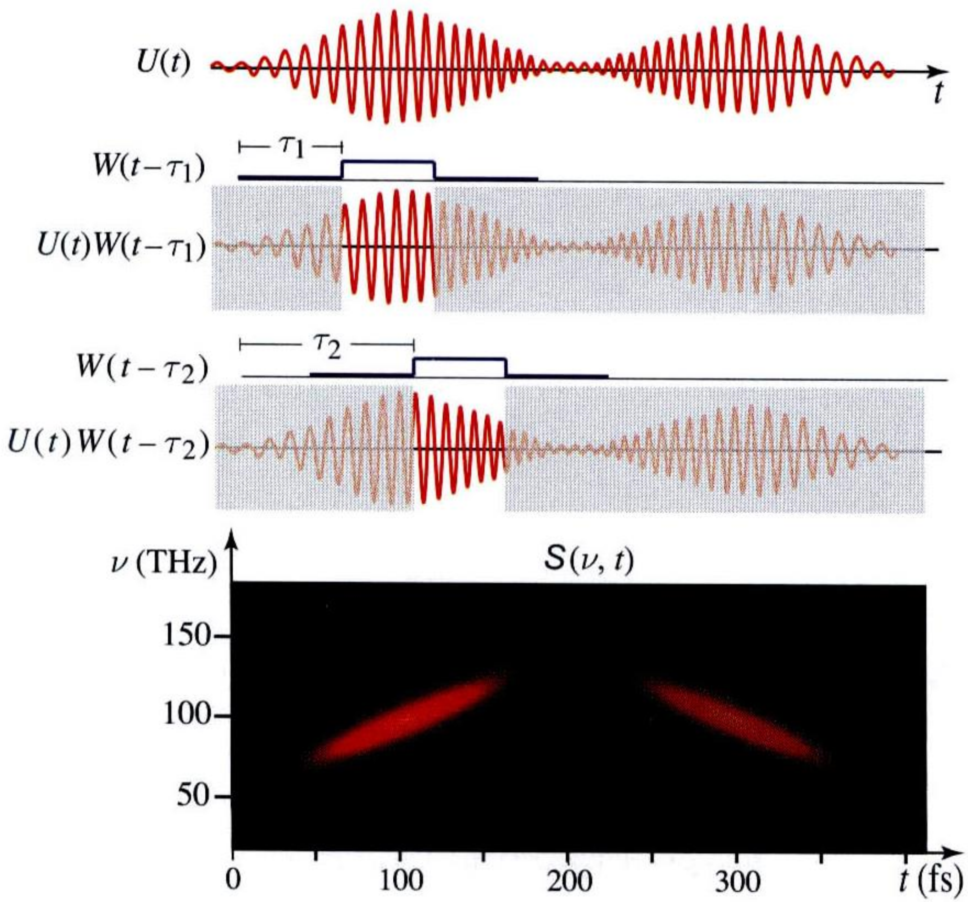
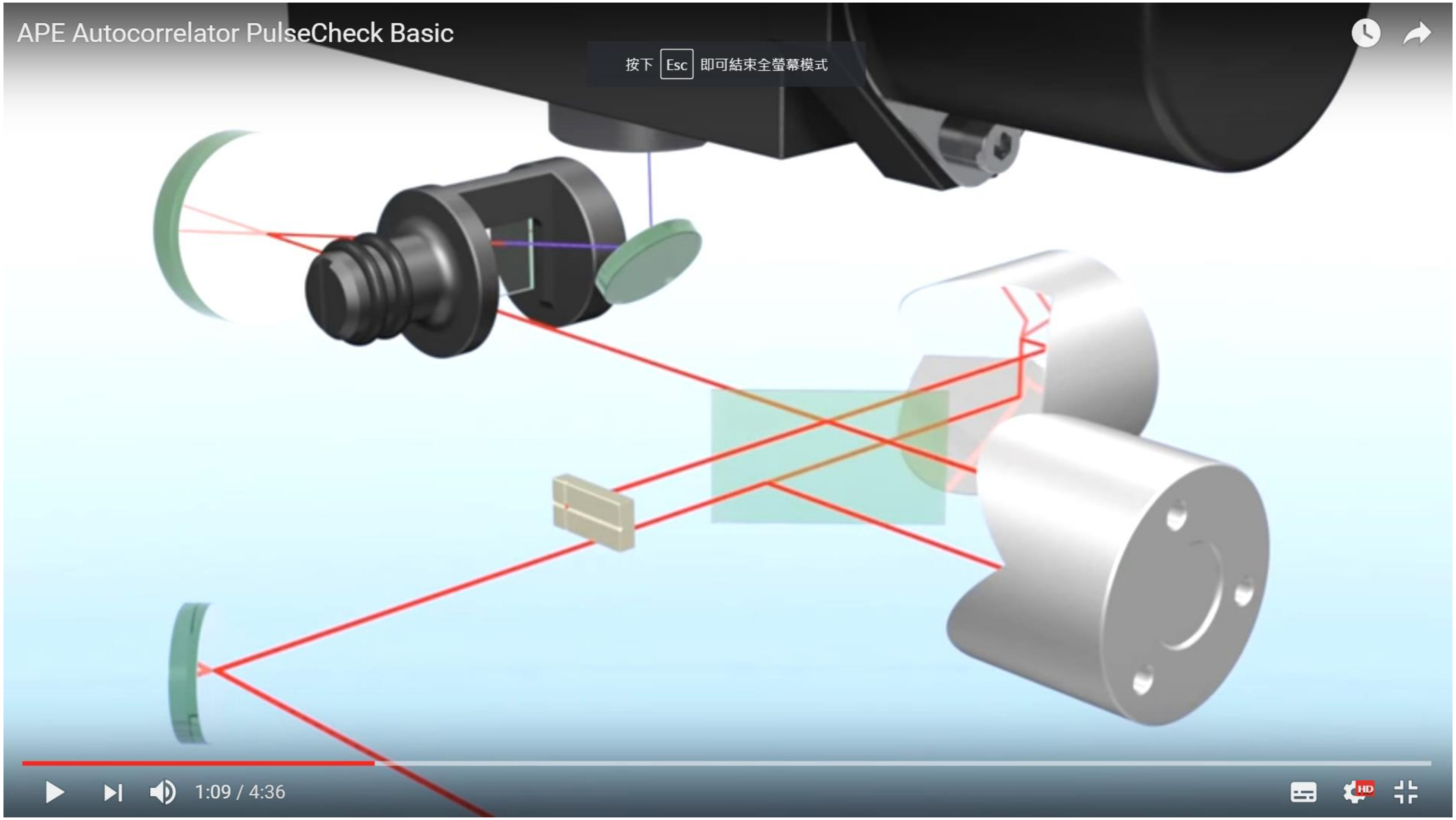


Figure 22.1-4 The short-time Fourier transform of $U(t)$ is constructed by a sequence of Fourier transforms of $U(t)$ multiplied by a moving window $W(t-\tau)$. The spectrogram $S(\nu, t)$ is the squared magnitude of these Fourier transforms. In this example, $U(t)$ is composed of two Gaussian pulses each of time constant $\tau = 60$ fs and central frequency 100 THz. The first pulse is up-chirped ($a = 5$) and the second is down-chirped ($a = -5$) and has a smaller amplitude. The window function $W(t)$ is Gaussian with time constant $\tau = 20$ fs.

Autocorrelator

- https://www.youtube.com/watch?v=J1pNHYySSYg



Transform-Limited Gaussian Pulse

- A **transform-limited Gaussian pulse** has constant **phase** and **Gaussian magnitude**.

$$A(t) = A_0 \exp(-t^2 / \tau^2)$$

  – **Intensity** : $I(t)=I_0\exp(-2t^2/\tau^2)$
  – τ_{FWHM} : 1.18τ
  – **Fourier transform of A(t):**

    $A(\nu)=A\exp(-\pi^2\tau^2\nu^2)$ (Gaussian function)

  – **Spectral intensity** : $S(\nu)=A^2\exp(-2\pi^2\tau^2\nu^2)$
  – $\Delta\nu=0.375/\tau=0.44/\tau_{FWHM}$ => $\Delta\nu\tau_{FWHM}=0.44$

- The **transform-limited** Gaussian pulse has a minimum **temporal**-and **spectral-width product**. (transform limited, Fourier-transform limited, bandwidth limited)

Transform-Limited Gaussian Pulse

$$A^2 \exp(-2\pi^2\tau^2\nu^2) = \frac{1}{2}$$

$$(-2\pi^2\tau^2\nu^2) = \ln\frac{1}{2} = -0.7$$

$$\sqrt{2}\pi\tau\nu = \sqrt{0.7}$$

$$\nu = \frac{0.1882}{\tau}$$

$$\Delta\nu = \frac{0.3764}{\tau}$$

$$I_0 \exp(-2t^2/\tau^2) = \frac{1}{2}$$

$$(-2t^2/\tau^2) = \ln\frac{1}{2} = -0.7$$

$$\sqrt{2}t/\tau = \sqrt{0.7}$$

$$t = 0.591\tau$$

$$\tau_{\mathrm{FWHM}} = 1.182\tau$$

- A more general **Gaussian pulse** :
  - $A(t)=A_0\exp(-\alpha t^2)$
  - **$\alpha = (1-ja)/\tau^2$** , "a" is the chirp parameter.

$$A(t) = A_0 \exp(-t^2/\tau^2)\exp(jat^2/\tau^2)$$

  - Up chirped : $a > 0$
  - Down chirped : $a < 0$
  - Unchirped (transform-limited) : $a = 0$
  - phase: $\varphi = at^2/\tau^2$ (quadratic function)
  - instantaneous frequency: $\nu_i = \nu_0 + at/\pi t^2$ (linear function of time)
  - The pulse is **linearly chirped** with chirp parameter a.
  - **Fourier transform of A(t):** $A(\nu)=A\exp(-\pi^2\tau^2\nu^2/\alpha)$
  - **Spectral intensity** : $S(\nu)=A^2\exp(-2\pi^2\tau^2(\nu-\nu_0)^2/(1+a^2))$
  - FWHM of Gaussian: $\Delta\nu=0.375/\tau\,(1+a^2)^{1/2}=0.44/\tau_{FWHM}$
  - $\tau_{FWHM}\Delta\nu = 0.44\mathbf{(1+a^2)^{1/2}}$

Temporal and spectral properties of a chirped Gaussian pulse of peak amplitude A_0, **peak intensity** $I_0 = |A_0|^2$, **central frequency** ν_0, time constant τ, and **chirp parameter** a.

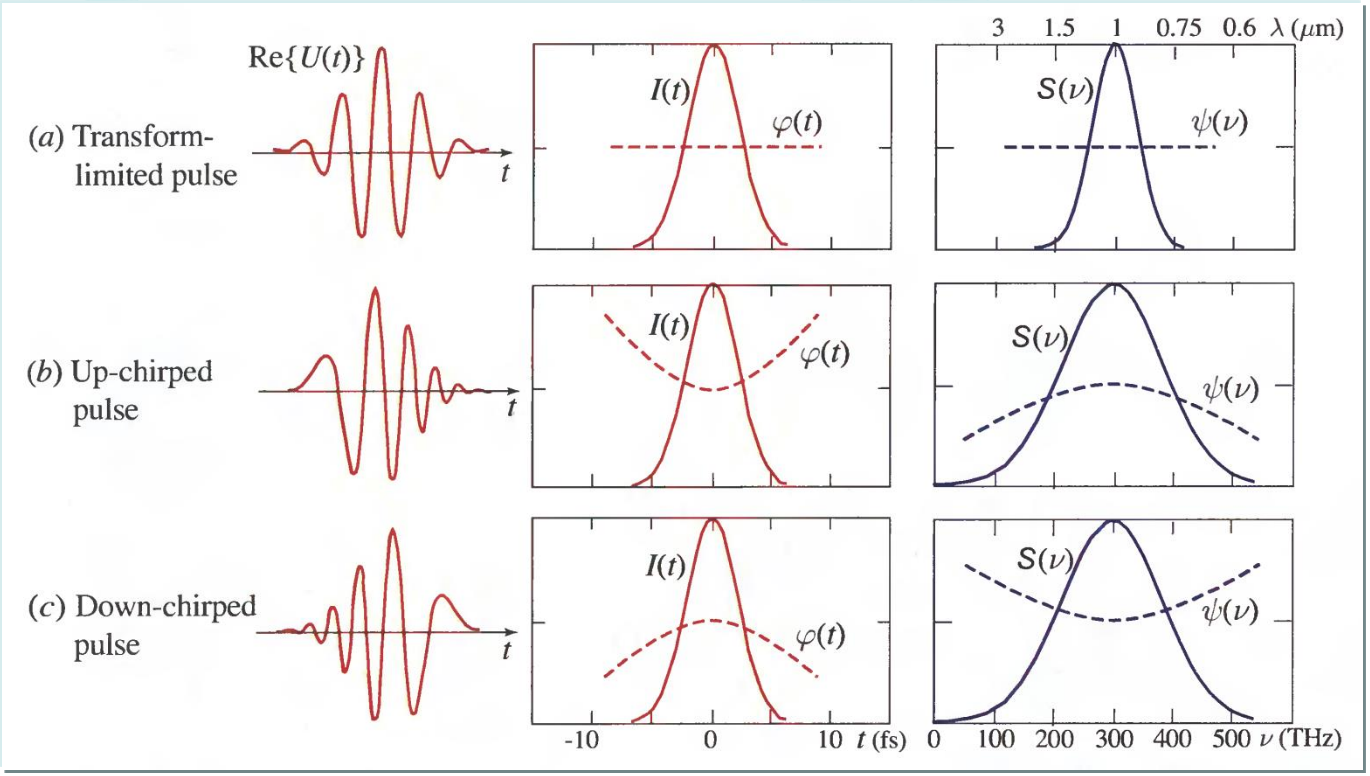
$\mathcal{A}(t) = A_0 \exp[-(1-ja)t^2/\tau^2]$	Complex envelope
$I(t) = I_0 \exp(-2t^2/\tau^2)$	Intensity
$\int I(t)dt = \sqrt{\pi/2} I_0 \tau$	Energy density
$\tau_{1/e} = \sqrt{2}\tau$	$1/e$ half width
$\tau_{\mathrm{FWHM}} = 1.18\tau$	FWHM width
$\varphi(t) = at^2/\tau^2$	Phase
$A(\nu) = \dfrac{A_0\tau}{2\sqrt{\pi(1-ja)}} \exp\left[-\dfrac{\pi^2\tau^2\nu^2}{1-ja}\right]$	Fourier transform
$S(\nu) = \dfrac{I_0\tau^2}{4\pi\sqrt{1+a^2}} \exp\left[-\dfrac{2\pi^2\tau^2(\nu-\nu_0)^2}{1+a^2}\right]$	Spectral intensity
$\Delta\nu_{1/e} = \dfrac{2}{\tau}\sqrt{1+a^2}$	$1/e$ half width
$\Delta\nu = \dfrac{0.375}{\tau}\sqrt{1+a^2} = \dfrac{0.44}{\tau_{\mathrm{FWHM}}}\sqrt{1+a^2}$	FWHM Spectral width
$\psi(\nu) = -2\pi^2\tau^2[a/(1+a^2)]\nu^2$	Spectral phase
$\nu_i = \nu_0 + (a/\pi\tau^2)t$	Instantaneous frequency

Temporal and **spectral profiles** of three Gaussian pulses of central frequency ν_0=300 THz (corresponding to a wavelength of 1 pm and a 3.3-fs optical cycle) and width 7 FWHM = 5 fs (7 = 4.23 fs).

(a) **Transform-limited pulse**; the **spectral width** Δν == 88 THz (Δλ = 73 nm).

(b) (b) **Up-chirped** pulse of chirp parameter *a* == 2; the spectral width is a factor of $(1 + a^2) = 5^{1/2}$ greater than in (a), so that Δν == 197 THz.

(c) (c) Same as in (b) but the pulse is down-chirped with chirp parameter a = -2.



OUTLINE

- 22.2 A Chirp Filter
  - Linear Filtering of an Optical Pulse
  - The Ideal Filter
  - The Chirp Filter
  - Approximation of Arbitrary Phase Filter by a Chirp Filter
  - Chirp Filtering of a Transform-Limited Gaussian Pulse
  - Chirp Filtering of a Chirped Gaussian Pulse

Linear Filtering of an Optical Pulse

- A **linear time-invariant system** is characterized by a transfer function $H(\nu)$
  - it is the factor by which the **Fourier component** of the **input pulse** at frequency ν is multiplied to generate the **output component** at the same frequency

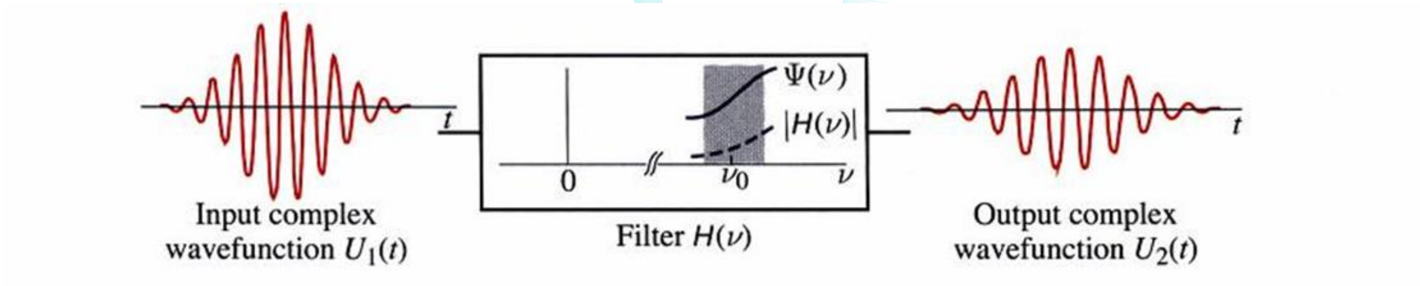
    $U_1(t)$: the **complex wavefunction** of the **original pulse**

    $U_2(t)$: the **complex wavefunction** of the **filtered pulse**

- The **Fourier transforms** $V_1(\nu)$ and $V_2(\nu)$ are related by :

$$V_2(\nu) = H(\nu)\, V_1(\nu). \tag{22.2-1}$$

- In using (22.2-1) we only need to know ***H*(ν)** at frequencies within **spectral band** of the pulse (a region of width $\boldsymbol{\Delta\nu}$ surrounding the central frequency ν_0)



$$V_2(\nu) = H(\nu)\,V_1(\nu). \qquad (22.2\text{-}1)$$

- When $\Delta\nu \ll \nu_0$ (it is convenient to work with the **complex envelope** instead of the **wavefunction)**:
- Using the relation:

$$U(t) = A(t)\exp(j2\pi\nu_o t)$$

- The shift property of the Fourier transform:

$$V(\nu) = A(\nu - \nu_0)$$

$A(\nu)$: the Fourier transform of $A(t)$

- It follows from (22.2-1) that

$$A_2(\nu - \nu_0) = H(\nu) A_1(\nu - \nu_0)$$

$$V_2(\nu) = \mathbf{H}(\nu)\, V_1(\nu). \qquad (22.2\text{-}1)$$

- Define: $f = \nu - \nu_0$

$$\Rightarrow \quad A_2(f) = H(\nu_0 + f)A_1(f)$$

$$\Rightarrow \quad A_2(f) = H_e(f)A_1(f)$$

- The envelope transfer function:

$$\boxed{H_e(f) = H(\nu_0 + f)} \qquad (22.2\text{-}3)$$

Envelope Transfer Function

- $H(\nu)$ (transfer function) : $H(\nu) = |H(\nu)| \exp[-j\psi(\nu)]$

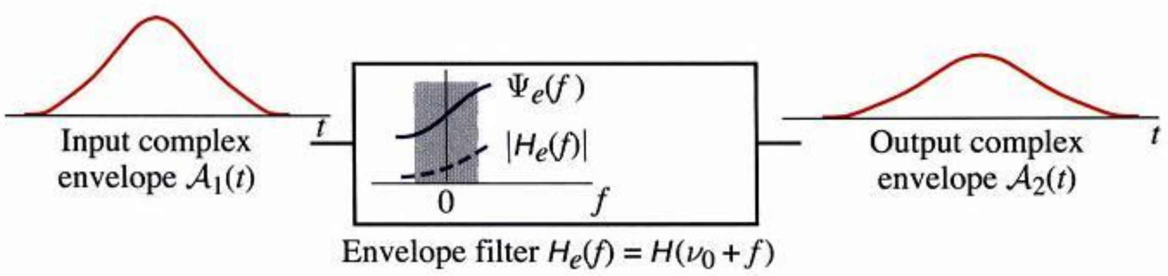
- $H_e(f)$ (transfer function) : $H_e(f) = |H_e(f)| \exp[-j\psi_e(f)]$

- Phase transfer $\psi_e(f) = \psi(\nu_0 + f)$

- The **filter** often plays a **more important** role than the **magnitude** in the **reshaping of pulses**
- Throughout this **chapter** we will deal **phase filters**
  - The magnitude $|\boldsymbol{H(v)}|$ is **approximately constant** within the frequency range of interest
- In **time domain**, (22.2-2) becomes the **convolution relation**

$$A_2(t) = \int_{-\infty}^{\infty} h_e(t-t')\, A_1(t')\, dt', \qquad (22.2\text{-}4)$$

$h_e(t)$: Inverse Fourier transform of $H_e(f)$



$$A_2(f) = H_e(f) A_1(f), \qquad (22.2\text{-}2)$$

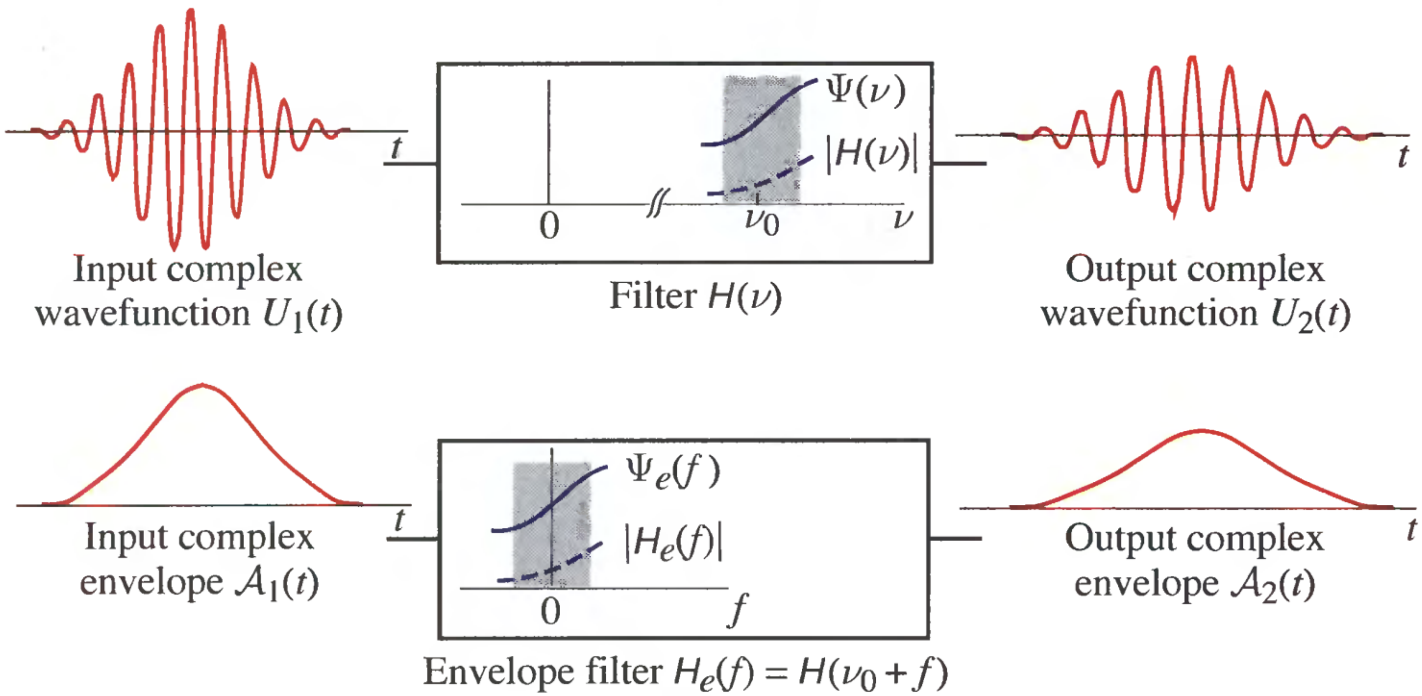


Figure 22.2-1 Filtering the wavefunction with a filter $H(\nu)$ (upper figure) is equivalent to filtering the envelope with a filter $H_e(f) = H(\nu_0 + f)$ (lower figure). The shaded area represents the spectral band of interest.

The Ideal Filter

An **ideal filter**

It preserves the **shape** of the **pulse envelope,** it merely multiplies it by a **constan**t(<1:**attanuatior**; >1:**gain**), and delays it by a **fixed time**.

- The **transfer function** has the form

$$H_e(f) = H_0 \exp\left(-j2\pi f \tau_d\right), \tag{22.2-5}$$

H_0 : a constant,

τ_d : time delay

$G=|H_0|^2$:the **intensity reduction** or **gain factor**

$$H_e(f) = |H_e(f)| \exp[-j\psi_e(f)]$$

- The **phase** is a **linear function** of frequency $\boldsymbol{\Psi_e(f)=\Psi_0+2\pi\tau_d f}$

$$\Psi_0 = arg\{H_0\}$$

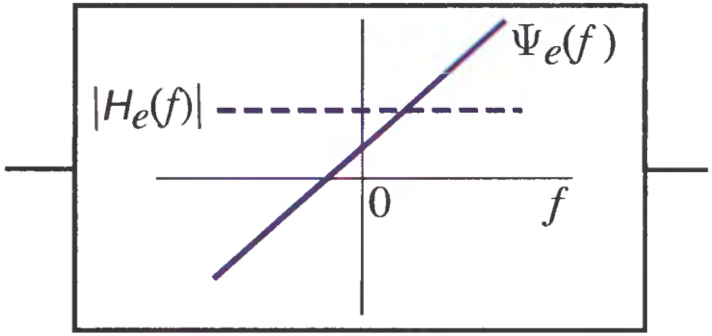
the **phase** $\boldsymbol{2\pi f \tau_d f}$ is equivalent to a **time delay** τ_d.

- The **input** and **output envelope** are related by

$$\boxed{\boldsymbol{A_2(t)=H_0 A_1(t-\tau_d)}}$$

- The **intensities** are related by

$$\boldsymbol{I_2(t)=GI_1(t-\tau_d)}$$



(a) Ideal filter

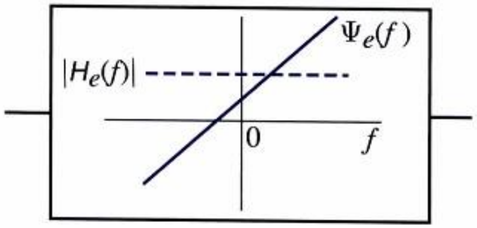
$$A_2(f) = H_e(f)A_1(f), \qquad (22.2\text{-}2)$$

$$H_e(f) = H_0 \exp(-j2\pi f \tau_d), \qquad (22.2\text{-}5)$$

- For a distributed **attenuator/amplifier** of **attenuation/gain coefficient** $\boldsymbol{\alpha}$ ($\tau_d = d/\mathrm{c}$, $H_0=\exp(-\alpha d/2)$)

$$H_e(f) = \exp(-\alpha d / 2)\exp(-j2\pi fd/c)$$

$$G = \exp(-\alpha d)$$



(*a*) Ideal filter

- A slab of **ideal nondispersive material** with
  - **attenuation coefficient** α
  - **refractive index** $\boldsymbol{n}$ ($c = c_0/n$)

$$H_e(f) = H_0 \exp(-j2\pi f\tau_d), \qquad (22.2\text{-}5)$$

- The **transfer function (β=2πν/c,** propagation constant**)**

$$H(\nu)=\exp(-\alpha d/2)\exp(-j\beta d)$$

$$H_e(f)=\exp(-\alpha d/2)\exp(-j2\pi fd/c)$$

$$\boxed{H_e(f) = H(\nu_0 + f)} \qquad (22.2\text{-}3)$$

Envelope Transfer Function

- When $\boldsymbol{\alpha}$ and $\boldsymbol{n}$ are frequency dependent , the medium is **divspresive**, i.e., the filter is **not ideal** and the pulse shape may be altered

The Chirp Filter

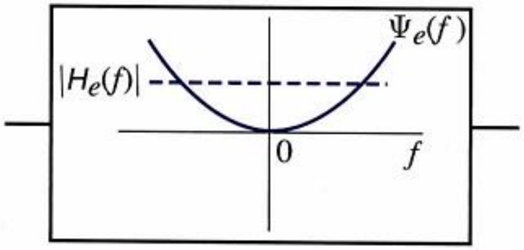
- The **Gaussian chirp filter** is the **most important filter** in **ultrafast optics,** we often simply called the **chirp filter**
- The **envelope transfer function** is Gaussian:

$$\boxed{H_e(f) = \exp\left(-jb\pi^2 f^2\right),} \qquad (22.2\text{-}6)$$

Chirp-Filter Transfer Function

b (real parameter, unit of s^2): **chirp coefficient** of the filter

- ***b*>0 :** the filter is **up-chirping**
- ***b*<0 :** the filter is **down-chirping**



(*b*) Chirp filter

- The corresponding **impulse-response function** is the inverse FT of (22.2-6) **(Gaussian function)**:

$$\boxed{h_e(t) = \frac{1}{\sqrt{j\pi b}} \exp(jt^2/b).} \qquad (22.2\text{-}7)$$

Chirp-Filter
Impulse-Response Function

- It too has a **phase** that is a **quadratic function** of time, i.e., it is a linearly chirped function,
  - **up-chirped** for positive b
  - **down-chirped** for negative b.

$$A_2(t) = \int_{-\infty}^{\infty} h_e(t-t')\,A_1(t')dt', \qquad (22.2\text{-}4)$$

- A cascade of **two chirp filters** with coefficients $\boldsymbol{b_1}$ and $\boldsymbol{b_2}$ is equivalent of single pulse with coefficient b**:**

$$b = b_1 + b_2$$

  - A **down-chirping filter** may compensate the effect of **an up-chirping filter**

- By substituting (22.2-7) into (22.2-4) , the **pulse envelope** at the **output** and **input of chirp filter** are related by:

$$\mathcal{A}_2(t) = \frac{1}{\sqrt{j\pi b}} \int_{-\infty}^{\infty} \mathcal{A}_1(t') \exp\left[j\frac{(t-t')^2}{b}\right] dt'. \qquad (22.2\text{-}8)$$

$$\boxed{\mathcal{A}_2(t) = \int_{-\infty}^{\infty} h_e(t-t')\,\mathcal{A}_1(t')dt', \qquad (22.2\text{-}4)}$$

$$\boxed{h_e(t) = \frac{1}{\sqrt{j\pi b}} \exp(jt^2/b).} \qquad (22.2\text{-}7)$$

Chirp-Filter
Impulse-Response Function

Chirp Filtering of a Transform-Limited Gaussian Pulse

- We now consider the effect of a **chirp filter** with **transfer function** (b: chirp coefficient)

The chirp filter

$$H_e(f) = e^{-jb\pi^2 f^2}$$

- An **unchirped Gaussian pulse** (transform limited)

$$A_1(t) = A_{10} e^{-t^2/\tau_1^2}$$

- **Fourier transform** of $A_1(t)$

$$A_1(f) = (A_{10}\tau_1 / 2\sqrt{\pi}) e^{-\pi^2 \tau_1^2 f^2}$$

- The **filtered pulse** has a **complex envelope** with FT

$$\boxed{A_2(f) = A_0 \frac{\tau_1}{2\sqrt{\pi}} \exp[-\pi^2(\tau_1^2 + jb) f^2]. \qquad (22.2\text{-}12)}$$

$$A_2(f) = A_0 \frac{\tau_1}{2\sqrt{\pi}} \exp[-\pi^2(\tau_1^2 + jb) f^2]. \qquad (22.2\text{-}12)$$

- This expression may be cast as the **Fourier transform** of a **chirped Gaussian pulse(width** t_2**, chirp parameter** a_2) (in accordance with (22.1-18))

$$A(\nu) = \frac{A_0 \tau}{2\sqrt{\pi(1-ja)}} \exp\left[-\frac{\pi^2 \tau^2 \nu^2}{1-ja}\right]$$

- Equating the phase, we obtain:

$$\tau_1^2 + jb = \frac{\tau_2^2}{1-ja_2}, \qquad (22.2\text{-}14)$$

- Equating the real and imaginary parts of (22.2-14) leads to the expressions that relate the parameters of the output pulse to those of the input pulse

$$A_{20} = A_{10}\sqrt{1-ja_2}\,\tau_1/\tau_2$$

Equating the **real** and **imaginary parts** of (22.2-14) leads to the expressions that relate the parameters of the output pulse to those of the input pulse:

Width	$\tau_2 = \tau_1\sqrt{1 + b^2/\tau_1^4}$,	(22.2-15)
Chirp parameter	$a_2 = b/\tau_1^2$,	(22.2-16)
Amplitude	$A_{20} = \dfrac{A_{10}}{\sqrt{1 + jb/\tau_1^2}}$.	(22.2-17)

$$\tau_1^2 + jb = \frac{\tau_2^2}{1 - j a_2} = \frac{\tau_2^2(1 + j a_2)}{1^2 + a_2^2}$$

chirp parameter

実

$$\Rightarrow \tau_1^2 = \frac{\tau_2^2}{1 + a_2^2}$$

width

$$\Rightarrow \tau_2^2 = \tau_1^2(1 + a_2^2)$$

$$\Rightarrow \tau_2 = \tau_1\sqrt{1 + b^2/\tau_1^2}$$

虛

$$b = \frac{\tau_2^2 a_2}{1 + a_2^2} = \frac{\tau_1^2\cancel{(1 + a_2^2)}a_2}{\cancel{1 + a_2^2}} = \tau_1^2 a_2$$

$$\Rightarrow a_2 = b/\tau_1^2$$

$$A_{20} = A_{10}\sqrt{1 - j a_2}\;\tau_1/\tau_2$$

Amplitude

$$= A_{10}\frac{\sqrt{1 - j a_2}}{\sqrt{1 + a_2^2}} = A_{10}\frac{\sqrt{1 - j b/\tau_1^2}}{\sqrt{1 + b^2/\tau_1^4}} = A_{10}\frac{1}{\sqrt{1 + j b/\tau_1^2}}$$

Chirp Filtering of a Transform-Limited Gaussian Pulse

- We conclude that upon transmission through a chirp filter

$$\tau_2 = \tau_1\sqrt{1+a_2^{\ 2}} = \tau_1\sqrt{1+b^2/\tau_1^{\ 4}}$$

- The **pulse width** is increased by a factor

$$\sqrt{1+a_2^{\ 2}} = \sqrt{1+b^2/\tau_1^{\ 4}}$$

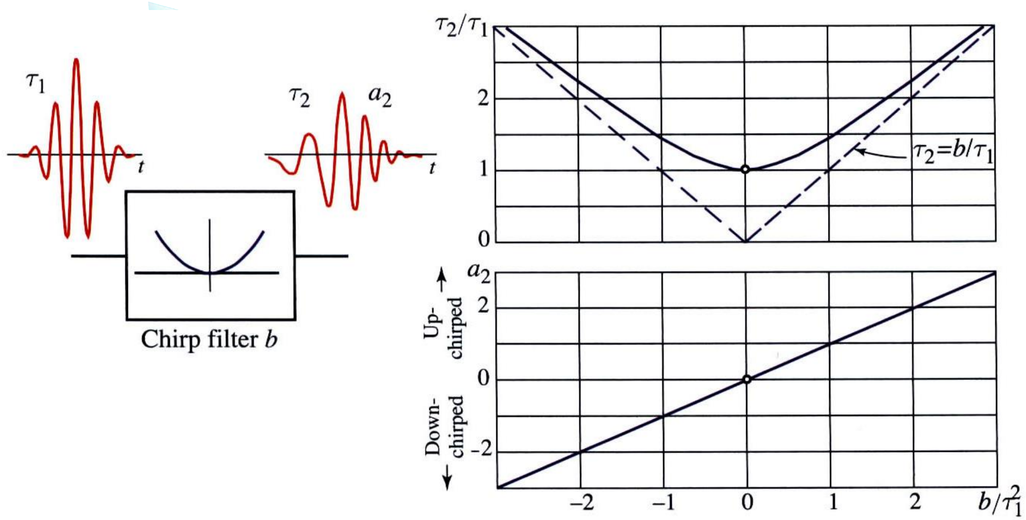
  – For $|b|=\tau_1^2$ this factor is $\sqrt{2}$
    - Thus, the **filter** begins to have a significant effect when its chip coefficient is of the order of the squared width of the original pulse.

- For $|b| \gg \tau_1^2 \Rightarrow \tau_2 \doteqdot |b|/\tau_1$,
  - indicating that the **width** of filtered pulse is **directly proportional to** $|b|$ and **inversely proportional to** τ_1 ,
  - **narrower pulses** undergo **greater broadening**

➢ The initially **transform-limited pulses** becomes chirped with a **chirp parameter** a_2 ($a_2 = b/\tau^2$)

- a_2 is **directly proportional** to the **filter chirp coefficient** b and **inversely proportional** to the square of τ_1 (the original pulse width)
- If b is **positive** => the filtered pulse is **up-chirped**
- If b is **negative** => the filtered pulse is **down-chirped**
- For $b = \tau_1^2$, the **chirp parameter** $a_2 = 1$.

- The spectral width of the pulse remains **unchanged**
  - the **chirp filter** is a **phase filter** that does not alter the **spectral intensity** of the original pulse.
  - The **temporal width** of pulse is expanded by a factor $\sqrt{1+a_2^{\,2}}$ => the **spectral width** must be compressed by the same factor
  - However, because the **filtered pulse** is **chirped** this is accompanied by a **spectral broadening** by the vary same factor, resulting in an **unchanged spectral width**



$$\tau_2 = \tau_1\sqrt{1+b^2/\tau_1^4},$$
$$a_2 = b/\tau_1^2,$$

Figure 22.2-4 A chirp filter with coefficient b converts an unchirped Gaussian pulse of width τ_1, marked by an open circle, into a chirped Gaussian pulse of width τ_2 and chirp parameter a_2. The pulse width increases as $|b|$ increases, and is greater for smaller τ_1 . The chirp parameter is directly proportional to b and is greater for smaller τ_1.

Chirp Filtering of a Chirped Gaussian Pulse

- When a **chirped Gaussian pulse** is transmitted through a **chirp filter**, the pulse will be **expanded** or **compressed** and its **chirp parameter** will be altered
  - and may under certain conditions **diminish to zero** so that the new pulse may become **unchirped (transform limited).**
- This compression property offers a technique for generation of picosecond and femtosecond optical pulses
- If the **original pulse** (**width**: τ_1, **chirp parameter**: a_1)

complex envelope: $A_1(t)=A_{10}\exp[-(1-ja_1)t^2/\tau_1^2]$,

chirp filter: $H_e(f)=\exp(-jb\pi^2 f^2)$

chirped Gaussian pulse: $A_2(t)=A_{20}\exp[-(1-ja_2)t^2/\tau_2^2]$

where

$$\frac{\tau_2^2}{1-ja_2}=\frac{\tau_1^2}{1-ja_1}+jb. \qquad (22.2\text{-}18)$$

- Equating the **real** and **imaginary** of (22.2-18) ,we obtain

$$\tau_2 = \tau_1 \sqrt{1 + 2a_1 \frac{b}{\tau_1^2} + (1 + a_1^2)\frac{b^2}{\tau_1^4}}\,, \qquad (22.2\text{-}19)$$

$$a_2 = a_1 + (1 + a_1^2)\frac{b}{\tau_1^2}\,. \qquad (22.2\text{-}20)$$

- To determine the value b_{min} of the **filter's chirp parameter** at which the **filtered pulse** has its **minimum width** τ_0
- We equate the **derivative of** τ_2 in (22.2-19) with respect to b to zero, the result is :

$$\boxed{\begin{array}{ll} \text{Minimum width} & \tau_0 = \dfrac{\tau_1}{\sqrt{1 + a_1^2}}\,, \\ \text{Chirp coefficient} & b_{\mathrm{min}} = -a_1 \tau_0^2 = -\dfrac{a_1}{1 + a_1^2}\tau_1^2. \end{array}} \qquad \begin{array}{l} (22.2\text{-}21) \\ (22.2\text{-}22) \end{array}$$

- Using (22.2-21) and (22.2-22) we rewrite (22.2-19) and (22.2-20) in terms of b_{min} and τ_0 as follows:

$$\text{Width} \quad \tau_2 = \tau_0 \sqrt{1 + (b - b_{\min}^2)/\tau_0^4}\,, \qquad (22.2\text{-}23)$$

$$\text{Chirp parameter} \quad a_2 = (b - b_{\min})/\tau_0^2. \qquad (22.2\text{-}24)$$

- when $b=b_{\min}$,(22.2-23) and (22.2-24) give $\tau_2=\tau_0$ and $a_2=0$ so that the pulse is both maximally compressed and unchirped
- If the original pulse is up-chirped ($a_1>0$) ,then $b_{min}<0$ so that down-chirping filter is necessary for maximal compression

$$\text{Minimum width} \quad \tau_0 = \frac{\tau_1}{\sqrt{1+a_1^2}}\,, \qquad (22.2\text{-}21)$$

$$\text{Chirp coefficient} \quad b_{\min} = -a_1\tau_0^2 = -\frac{a_1}{1+a_1^2}\tau_1^2. \qquad (22.2\text{-}22)$$

- Note that (22.2-23) and (22.2-24) are identical to (22.2-15) and (22.2-16), which were derived for the initially nchirped pulse, except that *b* is replaced by *b*-b_{min} .

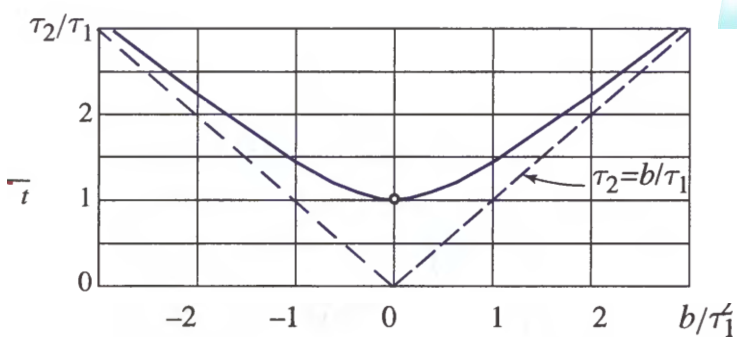
$$\text{Width} \quad \tau_2 = \tau_0\sqrt{1+(b-b_{\min}^2)/\tau_0^4}, \qquad (22.2\text{-}23)$$

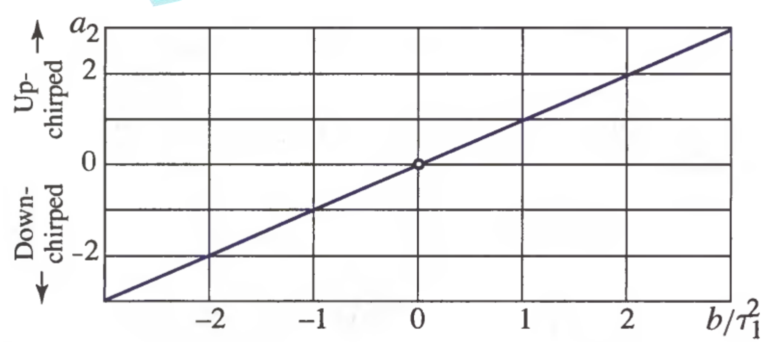
$$\text{Chirp parameter} \quad a_2 = (b-b_{\min})/\tau_0^2. \qquad (22.2\text{-}24)$$

$$\text{Width} \quad \tau_2 = \tau_1\sqrt{1+b^2/\tau_1^4}, \qquad (22.2\text{-}15)$$

$$\text{Chirp parameter} \quad a_2 = b/\tau_1^2, \qquad (22.2\text{-}16)$$

- Thus, the graphs in Fig. 22.2-4 are also applicable to the case of initially chirped pulse except for a shift in the horizontal direction by the value b_{min} determined from (22.2-22).





Application :Chirp Pulse Amplifer

- The **amplification** of an **ultrashort high-peak-power** optical is often limited by nonlinear effects (such as **saturation** and **self-focusing**)
- Such **limitations** may be alleviated if the pulse is **stretched** by use of **a chirp** filterfilter prior to amplification, and compressed by filtering through a **second chirp filter** after it has been amplified, as illustrated in Fig. 22.2-6.
- The **first filter** lowers the **peak power** by stretching the pulse
  - The **second chirp filte**r, which has a chirp parameter of equal magnitude and opposite sign => compresses the pulse back to its original width

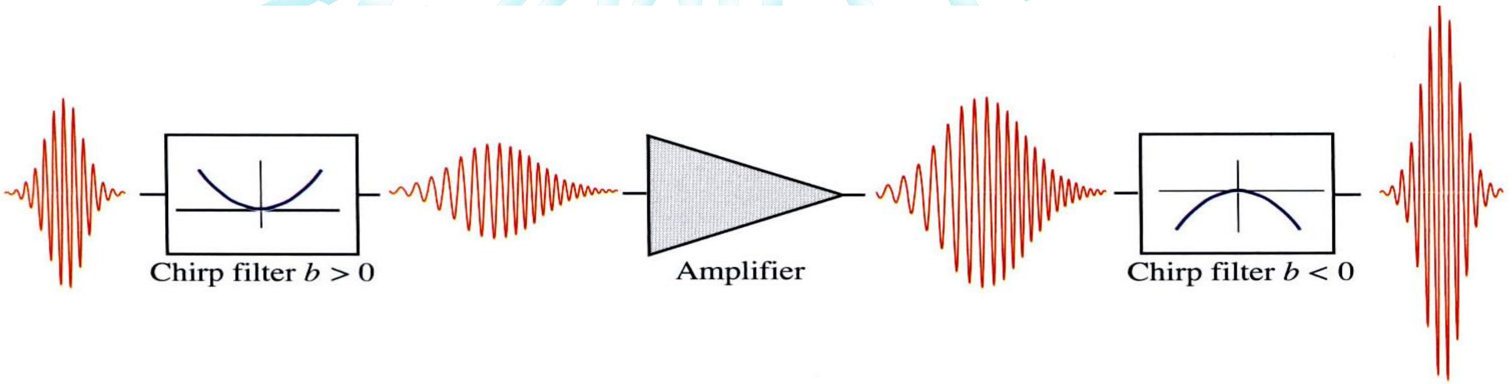


Figure 22.2-6 Chirp pulse amplifier.

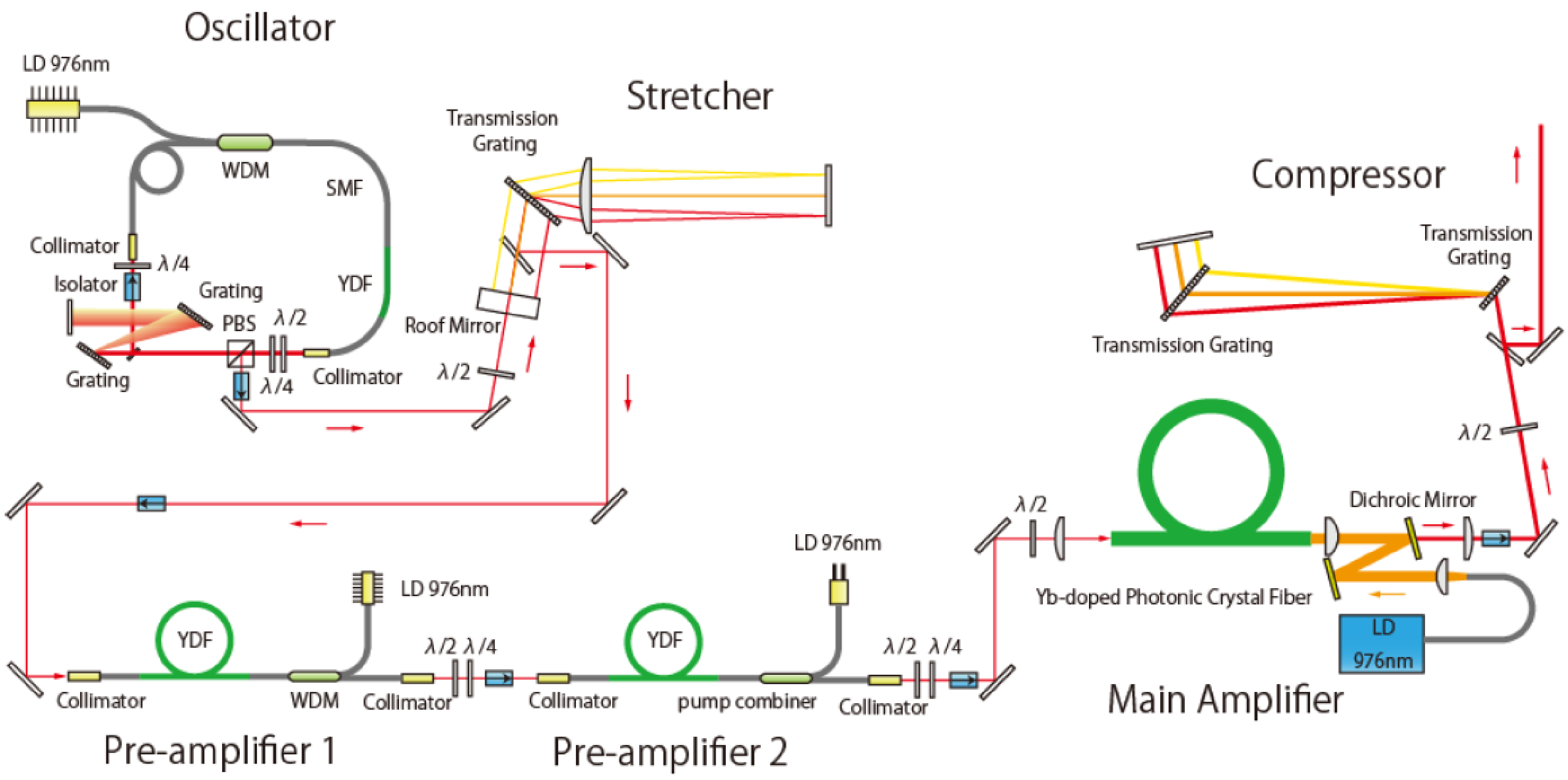


Fig. 1. Schematic of the experimental setup. WDM, wavelength division multiplexer; PBS, polarized beam splitter; SMF, single-mode fiber; LD, laser diode; λ/2, half waveplate; λ/4, quarter waveplate; YDF, Yb-doped fiber. Blue boxes with arrows are isolators.

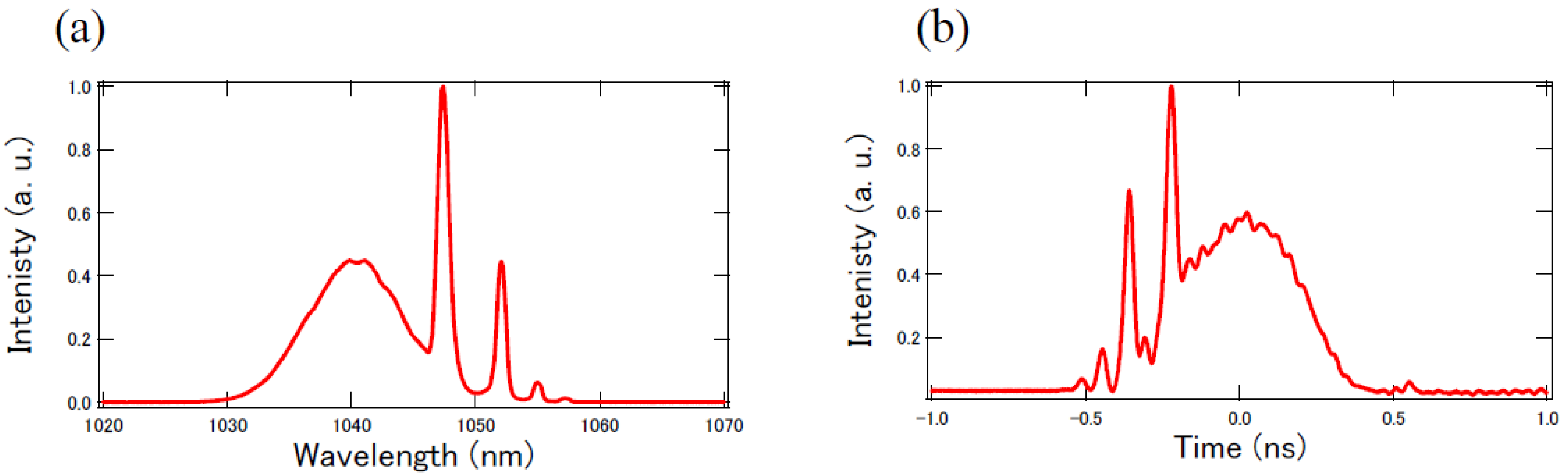


Fig. 3. (a) The spectrum after the stretcher. (b) The temporal pulse shape observed by a fast photo diode. The stretched pulse shape shows a clear chirp, whose structure corresponds to the spectral shape. The stretched pulse duration is 0.5 ns.

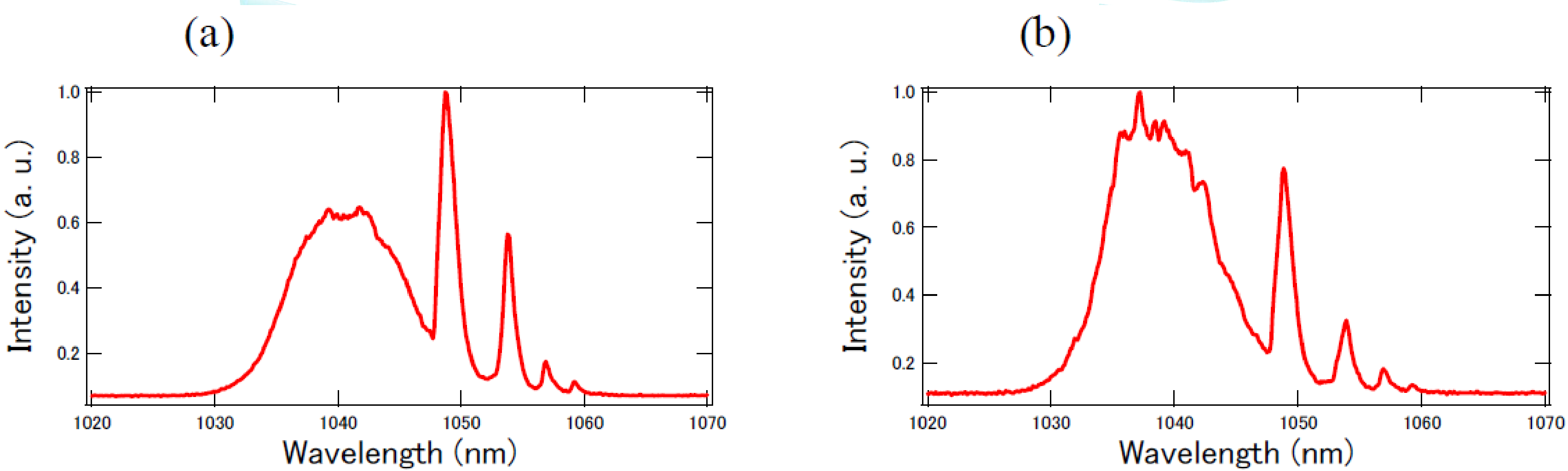


Fig. 4. Optical spectra measured after the (a) preamplifier, and (b) main amplifier.

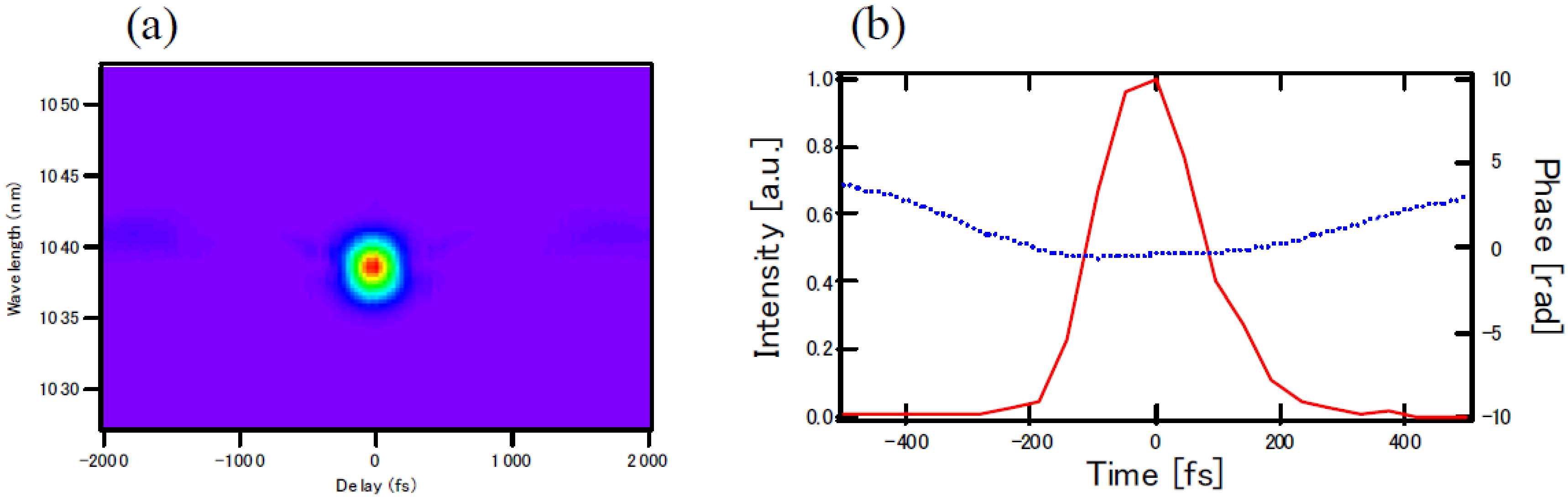
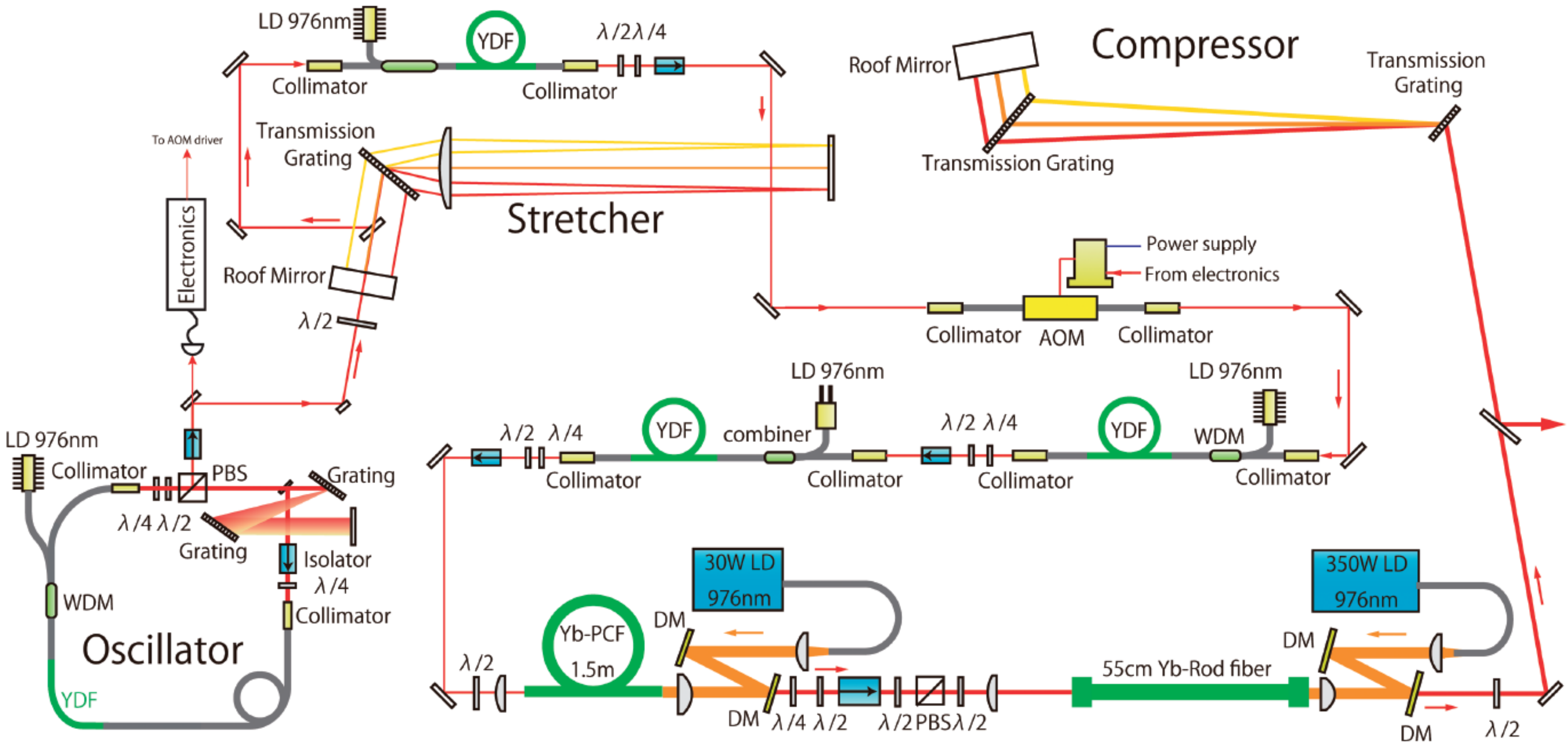
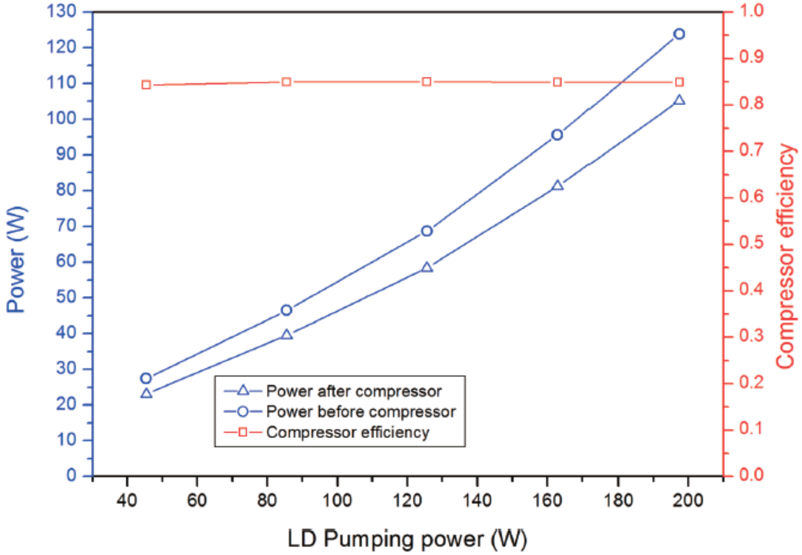


Fig. 5. (a) Observed FROG-trace with an average power of 23 W. (b) Retrieved temporal pulse shape and phase. This shows a pulse duration of 200 fs with no obvious pedestals.



LD 976nm
YDF
λ/2λ/4
Collimator
Collimator
Transmission Grating
To AOM driver
Electronics
Stretcher
Roof Mirror
λ/2
Compressor
Roof Mirror
Transmission Grating
Transmission Grating
Power supply
From electronics
Collimator
AOM
Collimator
LD 976nm
LD 976nm
λ/2 λ/4
YDF
combiner
λ/2 λ/4
YDF
WDM
Collimator
Collimator
Collimator
Collimator
Collimator
PBS
Grating
λ/4 λ/2
Grating
Isolator
λ/4
Collimator
WDM
Oscillator
YDF
30W LD 976nm
350W LD 976nm
Yb-PCF 1.5m
DM
λ/2
DM
λ/4 λ/2
λ/2 PBS λ/2
55cm Yb-Rod fiber
DM
DM
λ/2



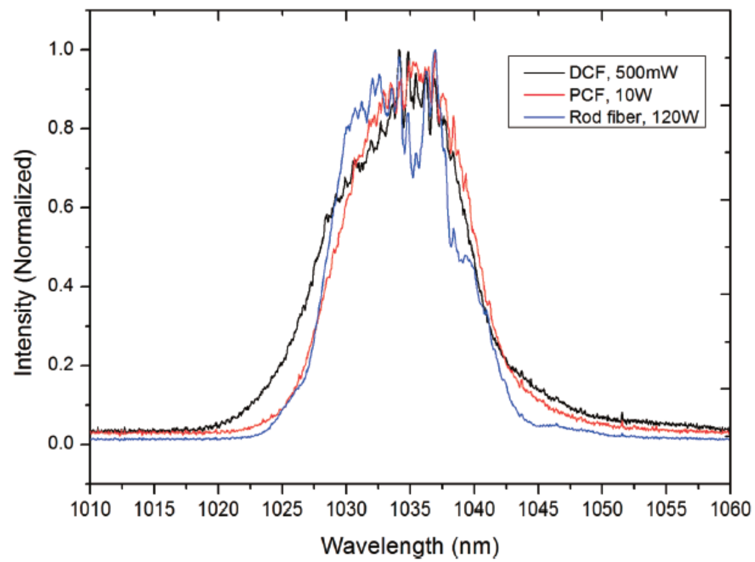
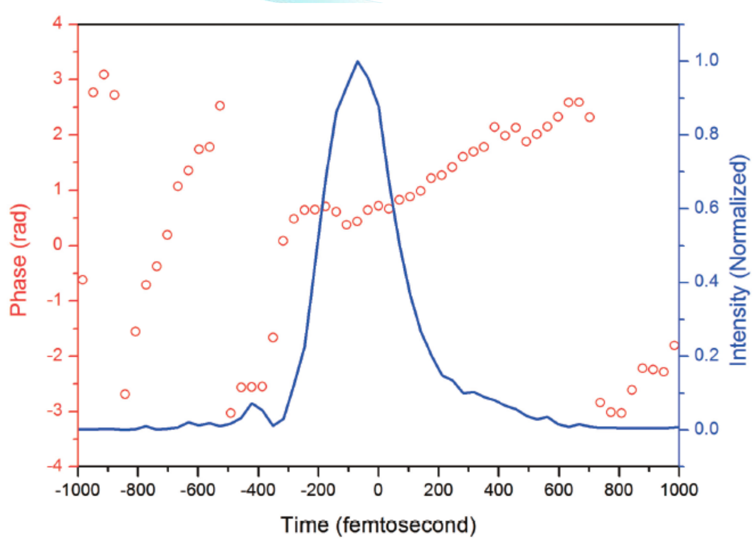


Fig. 2. Power performance of the 1 MHz fiber CPA laser system.

Fig. 3. Spectra recorded at different stages of amplification.



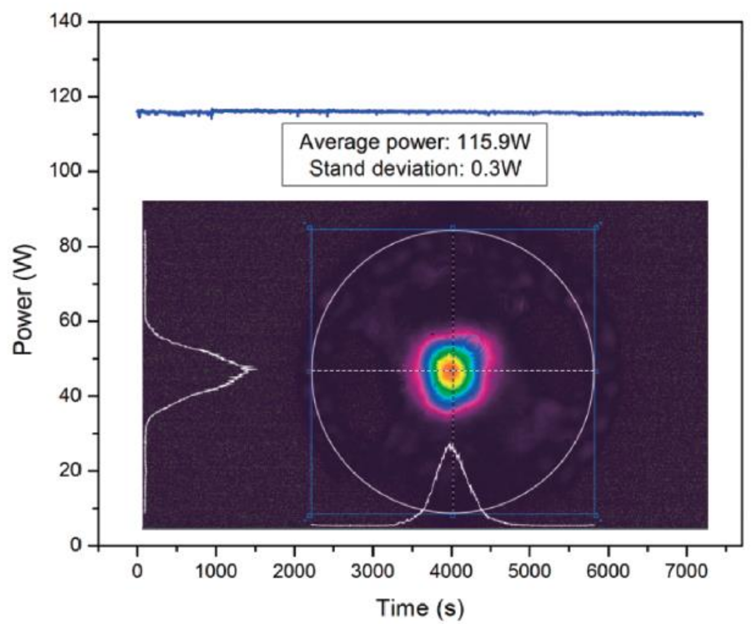


Fig. 4. Pulse intensity and phase obtained from the SHG-FROG measurement.